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# Intra-Firm Bargaining and Learning in a Market Equilibrium*

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***Abstract***

This paper introduces an agency relationship into a dynamic game with informational externalities. Two principals bargain with their respective agents about the production cost which is the private information of the agents and is correlated between them. We find that the agency relationship creates an incentive for simultaneous production, even if this involves an inefficient delay. As the commitment power of the principals decreases, this incentive becomes stronger. When principals compete, the effect of competition is decomposed into two parts. Inter-period competition (from past and future actions) pushes principals towards simultaneous actions, while intra-period competition (from concurrent actions) does the opposite.

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***Keywords***: bargaining, adverse selection, learning, information, externalities, delay.

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# 1 Introduction

When thinking about launching a new product, a firm's management or headquarters may not be completely sure about how much it will cost to develop. The R&D division usually has a better idea about the development costs. However, it is unlikely that the division would provide such information for free; typically, it would inflate the costs. The decision of a competitor to introduce a similar product will provide information that the product is not "too expensive" to develop. Headquarters now have a better bargaining position vis-à-vis their R&D division. However, the competitor's headquarters, which face the same problem, may also decide to wait for the information. Natural questions then arise. *When is the product launched? Do firms launch the product simultaneously? What is the effect of the strategic behavior of the R&D divisions? How does competition affect the equilibrium?*

The aim of this paper is to study the effect of the agency relationship in a dynamic game with informational externalities. Its crucial feature is that the information which is being released and learned is about strategic agents. These can influence what and when the uninformed parties (principals) learn. Agents' incentives are shaped by both current and expected future rents and, therefore, future information affects them even *before* it becomes known.

Learning about the agent from the experience of others but also providing information to them is a feature of many real-life situations. Consider a country opening up to foreign capital. When a foreign firm discusses a joint project (foreign direct investment) with a local firm, it does not know exactly what the cost will be. If local firms have similar technologies, the foreign firm can wait and have a better idea of local firms' characteristics by observing the experience of other foreign firms in this market. However, a late entry may not be as profitable as an early one. Another example is that of unions bargaining with firms. The firms are privately informed of their profits, their market prospects, etc. This information is correlated across firms in the same industry. Then, each union may delay the agreement in order to learn the outcome of negotiations in other firms. Waiting too long, however, can be costly as the suppliers and consumers may switch to other firms.

In our model, there are two identical principal-agent pairs, that we call firms, $A$ and $B$.[1] Each agent produces for his respective principal. The cost of each agent

[1]The agents can be thought of as independent (exclusive) suppliers in which case we have "vertical structures".

is his private information; principals know only the cost distribution. Each agent produces at most once during the relationship. The costs of the two agents are perfectly correlated.[2] For most of the paper, the principals do not compete, that is, the value they derive from the agent's production does not depend on the production (or its absence) of the other firm.

There are two periods, and all participants share a common discount factor. Inside each firm, the bargaining proceeds à la Sobel and Takahashi (1983). In period 1, each principal offers a price to her agent that she is ready to pay for the product in that period. If the agent accepts it, production takes place; the principal sells the good in the market, and the game ends for this firm. If the agent rejects the offer, the game moves to period 2. The principal then observes whether the other agent has produced the product or not and makes another offer to her agent that he is free to accept or reject.

In any equilibrium, there is a cut-off level of cost at which the agent is indifferent between accepting the first-period offer (and producing in period 1) and rejecting it (and waiting for a second-period offer). Thus in period 1, by his decision each agent reveals whether his cost is above or below this cut-off. Then, each principal has an incentive to delay the production in order to obtain information about the cost of the other agent and, because they are the same, the cost her own agent. However, the delay is costly because of discounting.[3]

The agency problem in our adverse selection environment means that each agent may pretend to have higher costs than he actually has. Then, he may reject a profitable offer in period 1 if he expects a sufficiently higher offer in period 2. In order to see the effect of this strategic behavior, we first consider the *benchmark setting* in which the agents are non-strategic: each of them accepts any offer in period 1 that is higher than his costs. We then study the *full-commitment setting* in which the agents are strategic (they can reject an offer above the costs in period 1), but the principals can commit to two-period contracts. We also look at the *no-commitment setting* in which the agents are strategic and the principals can propose only one-period contracts.

The only feature of the contracting in each firm that is relevant for the other principal is the information generated in period 1, i.e., the cut-off level of cost. We

[2] We consider imperfect correlation in Section 5.1.

[3] Competition between principals that we study in Section 3 may also make the delay costly.

look for the equilibria in these cut-offs. We find that there are only asymmetric equilibria (in pure strategies) in the benchmark setting, while there are only symmetric equilibria in the full and no-commitment settings. In the benchmark setting, having the same cut-offs implies that the two agents will generate exactly the same information. Principals prefer then to have different cut-offs in order to learn more. When the agents are strategic, in contrast, having the same cut-offs becomes useful as it puts the principals in the best position to limit the strategic behavior of the agents. If an agent deviates from the equilibrium behavior and rejects a first-period offer which is above his cost, the other agent will still accept the same offer and, therefore, the principal will detect the deviation and will not improve her offer in period 2.

More generally, we show that the sets of symmetric equilibria are always nested: the smallest is in the benchmark setting (it may be non-empty once competition is introduced) and the largest is in the no-commitment setting. In other words, the agency problem drives the two firms towards symmetric behavior. Moreover, when the contractual imperfections increase and only one-period contracts are available, there is more need to discipline the agents and the symmetric behavior becomes even more likely. The strategic behavior of the agents creates a coordination motive in principals' actions. Therefore, the internal structure of the firm and, in particular, the way the agency problem is solved cannot be studied in isolation from other firms in the market.

Efficiency is affected by informational externalities via two effects. The first effect is a standard one: the possibility to learn in the future causes the principals to delay the production, which harms efficiency.[4] The second effect is strategic and increases efficiency. Since offers in period 2 are made under better information, they give a lower rent to the agent in period 2 and, therefore, the agent becomes "softer", i.e., more likely to accept a given offer, in period 1. The second effect arises, obviously, only when the agents are strategic. In the benchmark setting there is just the first effect, so informational externalities decrease efficiency. In the full-commitment setting, the strategic effect is crucial. In the autarky, i.e., when there is only one principal-agent pair, the principal would commit to a take-it-or-leave-it offer in period 1 which is very inefficient.[5] Information obtained from the other firm

[4]We assume that principals make profits for any cost realization. Thus, any delay is inefficient and is due to rent extraction by the principals.

[5]This is a standard result in dynamic bargaining models. See, for example, Sobel and Takahashi

allows the principal to revise her offer and to contract with high cost types in period 2. In the no-commitment setting, the strategic effect becomes less important as the principals revise their offers anyway and the autarky is relatively efficient. The standard effect is likely to dominate, as in the benchmark setting.

Competition may take various forms and we allow for a great flexibility in the ways in which it affects principals' profits. We distinguish intra-period competition, that is, competition from concurrent sales, from inter-period competition, that is, competition from past sales and future sales. Competition from past sales reflects first-mover advantage: a second entrant in the market obtains smaller revenues than the first one. We find that inter-period competition pushes the two firms towards symmetric behavior. For example, in the benchmark setting the competitive pressure may force the principals to forego the benefits of more information and opt to make the same offers. The effect of intra-period competition is the opposite. Principals may choose to offer different contracts and tolerate some strategic play of the agents in order to differentiate themselves and produce in different periods.

The rest of the paper is organized as follows. Below we discuss the related literature. In Section 2 we introduce the model and find equilibria in the benchmark and the full-commitment settings assuming no competition. In Section 3 we introduce competition and analyze the effects of the four kinds of competition mentioned above. We study the no-commitment setting in Section 4. In Section 5 we consider alternative information structures. The first structure allows for imperfect correlation between the costs of the two agents, while in the second the costs of an agent are completely revealed if he produces in period 1, for example, through reverse engineering by the other principal. Section 6 concludes.

## 1.1 Related literature

The two most closely related papers are those of Gu and Kuhn (1998) and Drugov (2007). Gu and Kuhn (1998) study simultaneous bargaining of several firms with their unions. However, in order to simplify the model, they assume that the principal (union) can make an offer only once (but can choose when) and, therefore, the agent (firm) cannot be strategic.[6] This setup roughly corresponds to our benchmark

(1983) and Hart and Tirole (1988).

[6]In a related paper, Kuhn and Gu (1999) study the same game with the possibility of a strategic rejection, but impose a sequential order of moves and, therefore, the revelation of information is

setting. In Drugov (2007) the agent can strategically reject the principal's offer in an anticipation of a higher rent in the future. The model there is quite different as there are only two types of agent, but the quantity produced is continuous. In both papers, the only cost of the delay is discounting. In this paper, when principals compete, the cost of the delay is also a disadvantage of not being the first in the market and, therefore, depends on the strategy of other players. In other words, both the benefits of the delay (learning the information revealed by others) and the costs of the delay (being late in the market) are fully endogenous and may vary in different equilibria.

There is an extensive literature on learning from competitors. It may be about a technology (see, for example, Reinganum (1989) and De Bondt (1997) for surveys of patent, or R&D, races) or about the market or demand characteristics as in Rob (1991), Décamps and Mariotti (2004) and others. The crucial difference is that, in this paper, the learning is about the agent who is a strategic player. In models of the investment-under-uncertainty type as in Dixit and Pindyck (1994), better information in the future always delays the investment since it increases the payoff of investing in the future while the payoff of investing today is unchanged. In an agency model, better future information makes the agent softer both in the future and in the present, and may lead to more contracting in the present, i.e., a smaller delay (Drugov (2010), Fuchs and Skrzypacz (2010)).

Several recent papers such as Raith (2003), Golan, Parlour, and Rajan (2007) and Marin and Verdier (2008) study a market equilibrium in which the agency problem inside firms and market competition interact and influence each other. We also make a step in this direction. These papers study models with moral hazard in which competition affects contracts inside firms through changes in the profits that the principal obtains from a given agent's action. In our model, the agency problem is adverse selection, and competition has a dual role of providing information about the agents' types and changing principal's profits from a given contract. However, the informational role of competition may be important in the moral hazard environment as well. Indeed, starting with Lazear and Rosen (1981) and Holmström (1982) comparing performance of different agents is known to provide additional information, if there are common shocks to agents' performance.[7] Then, the dual role

exogenous. The first bargaining occurs in the autarky, while the second starts with more information.

[7]See also Meyer and Vickers (1997) for a dynamic model.

of competition seems natural independently of the type of the agency problem.

It is well known that the principal can use correlation among agents' types to extract rents (see Crémer and McLean (1985) and Shleifer (1985) for early papers). We obviously use this insight but the present paper differs in two aspects. First, in our paper there are two "principal-agent" pairs and not a single principal dealing with two agents. Second, we study a dynamic game which allows us to analyze delay in contracting.[8] Several papers study an adverse selection problem with an exogenous signal about agent's type. This is precisely the first step in analyzing our model, the signal being information revealed by the other firm (taken as given). Fuchs and Skrzypacz (2010) analyze an infinite-horizon bargaining model and show that the delay is non-monotonic in the likelihood of a signal that reduces informational asymmetry. In Drugov (2010), in a two-period model, we show a related result that the delay is non-monotonic in the quality of the signal (as Drugov (2007), it is a two-type model with continuous quantity). In Strausz (2006) the model is also dynamic but the agent has unlimited liability which is known to be crucial for Crémer-McLean mechanisms (together with risk neutrality). Other papers, such as Riordan and Sappington (1988), Demougin and Garvie (1991), Boyer and Laffont (2003) and Gary-Bobo and Speigel (2006), study a static adverse selection problem with an exogenous contractible signal.

# 2 The Model

## 2.1 Setup

There are two principal-agent pairs, that we call firms, $A$ and $B$, and in each of them the agent is supposed to produce a certain good. In each firm, the principal negotiates with her agent about the price she will pay for the production. There are two periods, and all parties share a common discount factor $\delta < 1$. Each agent produces at most once, either in period 1 or in period 2 or does not produce at all. The two agents have the same cost $\theta$ which is uniformly distributed on $[0, 1]$.[9,10] They know $\theta$, but principals know only its distribution.

[8] Another early paper is Nalebuff and Stiglitz (1983) that discusses, mostly informally, the benefits of relative compensation in markets where firms have correlated costs.

[9] Assuming uniform distribution allows us to obtain closed-form solutions. Qualitative results do not depend on this assumption.

[10] We consider imperfect correlation in Section 5.1.

If an agent produces the good in period 1, the principal sells it in the market and obtains revenues of $v_1$; producing in period 2 yields revenues $v_2$. In this Section, there is no competition between principals so the revenues do not depend on the behavior of the other firm (competition is analyzed in Section 3). Producing in period 1 is more profitable, $v_1 \geq v_2$, since the good may be sold for two periods (if it is non-durable) or it may be used for two periods (if it is a durable good). Also, it is always efficient to produce, $v_2 \geq 1$ . Therefore, if some types do not produce, this is only for rent-extraction reasons. Finally, we also assume that $v_1 \leq 2$ in order to avoid trivial cases.

The timing of the game is as follows. At the beginning of period 1 each principal proposes a contract to her respective agent. If an agent agrees to produce in period 1, he does so, the principal earns $v_1$ and the game is over for that firm. If the agent does not produce in period 1, the principal observes whether the other agent has produced in period 1 (but she does not observe the contract between the other principal and the agent). Then the principal makes an offer in the beginning of period 2 which the agent accepts or rejects. Production, transfers and selling of the good in the market, if any, take place and the game ends.

The agency problem in our adverse selection environment means that the agent may pretend to have higher costs than he actually has. Then, he may reject a profitable offer in period 1 if he expects a sufficiently higher offer in period 2. In order to see the effect of this strategic behavior, we first consider the *benchmark setting* in which the agents are non-strategic: they accept any offer in period 1 that is higher than their costs. It can also be interpreted as the principals committing to outsource the production if the offer is rejected. We then study the *full-commitment setting* in which the agents are strategic (they can reject an offer above their costs in period 1) and the principals can commit to a two-period contract (possibly conditional on the production of the other firm in period 1). In Section 4, we also look at the *no-commitment setting* in which the agents are strategic and the principals can propose only one-period contracts.

A strategy of each agent is a triple $(p_1, p_2^y, p_2^n)$ (all offered in period 1 or not, depending on the setting), where $p_1$ is the price to be paid to the agent if he produces in period 1, and $p_2^y$ and $p_2^n$ are the prices, conditional on whether the other firm has produced or not, to be paid for the production in period 2.[11] We consider only pure

[11] The fact that $p_1$ is not conditional on the behaviour of the other pair matters only if the agent

strategies.[12]

The equilibrium concept is Perfect Bayesian Equilibrium. We solve the model in the following way. In period 1, an agent is indifferent about producing or not producing when his type is the cut-off type, $\theta = s$. If $\theta < s$, the agent strictly prefers to produce in period 1, while he strictly prefers not to produce if $\theta > s$. The information revealed by each firm after period 1 is a partition, whether the type is below or above $s$. Then, $s$ summarizes all the information revealed by each firm after period 1 and, therefore, only $s$ (which is the probability of production in period 1) is payoff-relevant for the other principal. In an equilibrium, any strategy $(p_1, p_2^y, p_2^n)$ maps into a cut-off type $s$ of that firm's agent. The strategies of the two firms can then be characterized by reaction functions $s^A\left(s^B\right)$ and $s^B\left(s^A\right)$ and their intersections are equilibria. Since the two firms are (ex ante) identical, $s^A\left(s^B\right)$ and $s^B\left(s^A\right)$ are symmetric about the diagonal.

In each setting, we will first find the reaction function $s^A\left(s^B\right)$ (that is, we solve the problem of principal $A$) and then look for intersections with its inverse. We also find prices $p_1, p_2^y$ and $p_2^n$ offered by principal $A$ to agent $A$ (that we do not mark with superscript "$A$" to keep notation simple as this does not create any confusion.)

## 2.2 The Benchmark

In the benchmark setting, the agents do not behave strategically. Facing an offer $p_1$, agent $A$ accepts it if it is higher than his costs. Thus, the cut-off type is $s^A = p_1$.

After period 1, observing whether firm $B$ has produced or not, principal $A$ learns if the agent's costs $\theta$ are higher or lower than $s^B$ (remember that the costs are perfectly correlated). There are two cases to consider depending on whether their first-period offer $p_1$ is higher or lower than $s^B$.

Start with the case $s^A \geq s^B$ that we call *case a* (for *above*). The information generated by firm $B$ is useless since, following a rejection of an offer in period 1, the principal learns that $\theta$ is higher than $s^A$ which is more precise than the fact that $\theta$ is higher than $s^B$. Then, principal $A$ acts as if she were in the *autarky* (i.e., alone in

is strategic and it is justified by additional assumptions that we introduce in Section 2.3 on full commitment.

[12] Mixed strategies introduce a significant complication since the updated beliefs after observing (no) production by the other pair in period 1 are not truncated uniform. Also, a pure-strategy equilibrium always exists in our model.

the market) and solves the following problem[13]

$$\begin{cases} \max\limits_{p_1, p_2^n} \Pi_a^{BM} = (v_1 - p_1)\, p_1 + \delta\,(v_2 - p_2^n)\,(p_2^n - p_1) \\ \\ \text{s.t.} \quad s^B \leq p_1 \leq p_2^n \leq 1 \end{cases} \tag{1}$$

(superscript "BM" refers to the benchmark setting).

When $p_1$ and $p_2^n$ are offered, with probability $p_1 = s^A$ the production takes place in period 1, in which case the profits are $v_1 - p_1$; with probability $p_2^n - p_1$ it takes place in period 2, in which case the profits are $\delta\,(v_2 - p_2^n)$. If agent $A$ rejects $p_1$, principal $A$ never observes production in firm $B$ and therefore $p_2^y$ is irrelevant. The two prices are obtained from the first-order conditions of (1) unless the constraints $s^B \leq p_1, p_2^n \leq 1$ bind, that is,

$$p_1 = \min\left\{\max\left\{s^B, \frac{v_1 - \delta v_2 + \delta p_2^n}{2}\right\}, 1\right\}, \tag{2a}$$

$$p_2^n = \min\left\{\frac{v_2 + p_1}{2}, 1\right\}. \tag{2b}$$

Consider now the case $s^A \leq s^B$ which we call *case b* (for *below*). In this case the information generated by firm $B$ is useful since, following a rejection of a first-period offer $p_1 = s^A$, principal $A$ learns if the costs of the agent are in the interval $(s^A, s^B]$ or $(s^B, 1]$. Principal $A$ maximizes

$$\begin{cases} \max\limits_{p_1, p_2^y, p_2^n} \Pi_b^{BM} = (v_1 - p_1)\, p_1 + \delta\left[(v_2 - p_2^y)\,(p_2^y - p_1) + (v_2 - p_2^n)\left(p_2^n - s^B\right)\right] \\ \\ \text{s.t.} \quad p_1 \leq p_2^y \leq s^B \leq p_2^n \leq 1. \end{cases} \tag{3}$$

From the first-order conditions and the constraints, the expressions for the three prices are

[13] This is the setting of Sobel and Takahashi (1983).

$$
\begin{aligned}
p_1 &= \min\left\{\frac{v_1-\delta v_2+\delta p_2^y}{2}, s^B\right\}, &(4a)\\
p_2^y &= \min\left\{\frac{v_2+p_1}{2}, s^B\right\}, &(4b)\\
p_2^n &= \min\left\{\frac{v_2+s^B}{2}, 1\right\}. &(4c)
\end{aligned}
$$

The price offered when there was no production by firm $B$, $p_2^n$, does not depend on $p_1$ since the updated beliefs of the principal are that $\theta \in (s^B, 1]$, and therefore do not depend on $p_1$.

Once the maximal profits $\Pi_a^{BM}$ and $\Pi_b^{BM}$ have been found using (2) and (4), respectively, we can find when $s^A$ is above and below $s^B$ and therefore characterize the optimal contract as a function of $s^B$. This is done in the next Lemma. We only characterize $s^A$ in the text of the Lemma since it is enough to find equilibria. We give the complete characterization in the proof in the Appendix.

**Lemma 1** *In the benchmark setting, there are two cases:*

1. *If* $\frac{v_1+(2-\delta)v_2}{4-\delta} \geq 1$, *there exists* $\theta_1^{BM} \in \left(\frac{v_1-\delta v_2}{2-\delta}, \frac{v_1-\delta v_2+\delta}{2}\right)$ *such that*

$$
s^A = \left[\begin{array}{ll} \frac{v_1-\delta v_2+\delta}{2}, & \textit{if } s^B \leq \theta_1^{BM} \\ \frac{v_1-\delta v_2+\delta s^B}{2}, & \textit{if } s^B > \theta_1^{BM} \end{array}\right.
$$

2. *If* $\frac{v_1+(2-\delta)v_2}{4-\delta} < 1$, *there exists* $\theta_2^{BM} \in \left(\frac{v_1-\delta v_2}{2-\delta}, \frac{v_1-\frac{\delta}{2}v_2}{2-\frac{\delta}{2}}\right)$ *such that*

$$
s^A = \left[\begin{array}{ll} \frac{v_1-\frac{\delta}{2}v_2}{2-\frac{\delta}{2}}, & \textit{if } s^B \leq \theta_2^{BM} \\ \frac{v_1-\delta v_2+\delta s^B}{2}, & \textit{if } s^B \in \left[\theta_2^{BM}, \frac{v_1+(2-\delta)v_2}{4-\delta}\right] \\ \frac{v_1-\frac{\delta}{2}v_2}{2-\frac{\delta}{2}}, & \textit{if } s^B \geq \frac{v_1+(2-\delta)v_2}{4-\delta} \end{array}\right.
$$

**Proof.** See Appendix. ■

An immediate corollary of Lemma 1 is the following.

**Corollary 1** *In the benchmark setting,* $s^A$ *is never equal to* $s^B$.

The two cases are very similar.[14] When $s^B$ is small, $s^A$ is larger than $s^B$ and constant. We are in case $a$ and the information is not used. At a higher $s^B = \theta_1^{BM}$ or $s^B = \theta_2^{BM}$, depending on case 1 or 2, $s^A$ jumps down to $\frac{v_1 - \delta v_2 + \delta s^B}{2} < s^B$ (case $b$) and continues to be smaller than $s^B$ for even higher $s^B$. See Figure 1 for a numerical example. The most important feature of the optimal $s^A$ is that it never equals $s^B$, as Corollary 1 says. Indeed, whenever $s^A = s^B$, it generates exactly the same partition of the agent's type space and, therefore, the information from the other firm is useless. The principal then prefers either to decrease $s^A$ to be able to use the information from firm $B$ or to increase $s^A$ up to the autarky level. Also note that $s^A$ is the same at $s^B = 0$ and at $s^B = 1$ since there is no information actually revealed and, therefore, the autarky solution is implemented.

The two firms are identical; thus, the reaction function of firm $B$ is the inverse of that of firm $A$. Then, in order to find equilibria we need to find intersections of the reaction curve $s^A\left(s^B\right)$ with its inverse, which is done in the next Proposition (the exact values of equilibrium points are given in the Appendix).

**Proposition 1** *In the benchmark setting, there are only two equilibria and they are asymmetric.*

**Proof.** See Appendix. ■

In an asymmetric equilibrium, by definition, the first-period production in one firm, say, firm $A$, is higher than that in firm $B$, $s^A > s^B$. Then, when $\theta \in \left(s^B, s^A\right)$ only firm $A$ produces in period 1. In other words, with probability $s^A - s^B$, firm $A$ is the "leader" and firm $B$ is the "follower" producing in period 2 with a positive probability. The strategy of firm $B$ can be described as "wait and see" and it is more profitable than being the "leader" (see profits in Figure 1). Firm $A$, knowing that firm $B$ will wait, has to act on its own and use the autarky solution. This asymmetric equilibrium exists despite the fact that the firms are ex ante identical and there is a place for both of them in the market.

Let us discuss the efficiency of the equilibria. Production is always efficient in our model since $v_1 \geq v_2 \geq 1$. For the same total production over two periods, it is more efficient to produce in period 1 than in period 2 since $v_1 - \theta > \delta\left(v_2 - \theta\right)$. Second-period prices and, therefore, second-period production are weakly increasing

[14] The only difference between the two cases is that the constraint $p_2^n \leq 1$ always binds in case 1, while in case 2 it does not bind when $s^B < \theta_2^{BM}$ and $s^B > \frac{v_1 + (2-\delta)v_2}{4-\delta}$.

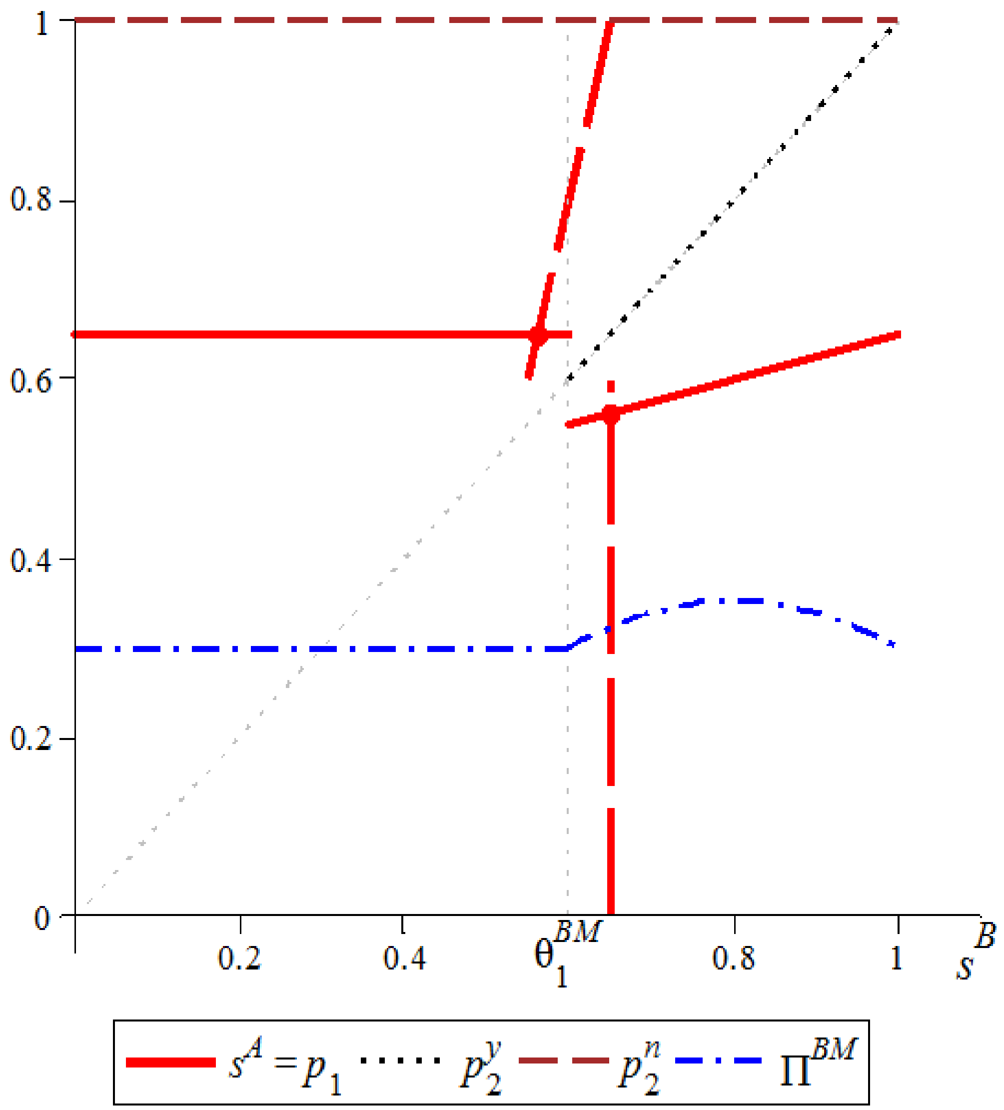


Figure 1: Prices and profits in the benchmark setting (case 1), $v_1 = 1.5$, $v_2 = 1.4$, $\delta = 0.5$. The long-dashed line is the inverse $p_1$ and its intersections with $p_1$ are (asymmetric) equilibria. For profits, the scale is adjusted.

in the first-period price (production). Then, efficiency increases with the first-period production, i.e., with $s$.

In an asymmetric equilibrium, the "leader" has a higher first-period production and, thus, is more efficient than the "follower". The behavior of the "leader", as we noted above, is the same as in the autarky, that is, when each firm acts on its own. The autarky is then more efficient since both firms would behave as "leaders". Indeed, informational externalities create an incentive to delay the production which is inefficient, though, privately optimal. This is the only effect in the benchmark setting.[15]

## 2.3 Full commitment

In this Section, the agents are strategic: they agree to produce in period 1 only if their rents are higher than their expected rent from waiting for period 2. The principals can commit to two-period contracts.

We make two following assumptions to ensure that, if principals want to use the information from the other firm, they have to delay the production until period 2.[16] First, limited liability of the agents: the rent earned by an agent (payment he receives minus his true costs) is consumed in the same period and cannot be claimed back later even if the principal realizes, after observing the other firm, that she has paid too much. Second, the credit (or cash) constraint of the agent: the principal has to cover the (claimed) costs of the agent in the period when the production takes place. That is, the principal cannot order the production and pay for it later when there will be more information.[17] These constraints imply that, at the beginning of period 1, each principal proposes a first-period price $p_1$ and a non-negative bonus to be paid later (conditional on future information). Moreover, only an agent with costs below $p_1$ can accept this offer, and therefore, $s^A \leq p_1$. The bonus could be used to bring $s^A$ closer to $p_1$; however, as we will see below, the full commitment solution involves $s^A = p_1$ and the optimal bonus is actually zero. Then, the contract

[15] In a different model, informational externalities could improve second-period production of all the firms, not only of the "follower". Then, there would be a trade-off between a higher delay of the "follower" and a better second-period production of the "leader".

[16] We do so that the full-commitment setting is more comparable to the benchmark setting of Section 2.2 and the no-commitment setting of Section 4.

[17] Alternatively, we could replace these two assumptions by assuming that with a small probability the costs of the two agents are independent and the agent can never incur a loss. See Section 5.1 where we solve such a model (for any probability that the costs are independent).

offered by each principal at the beginning of period 1 is a triple $(p_1, p_2^y, p_2^n)$.

Once contracts have been offered (simultaneously and secretly) by the principals, each agent decides whether to produce in period 1 or not; if he does not, he will be offered a pre-specified second-period price conditional on the production of the other firm. As in the benchmark setting, we first look for the optimal cut-off type of agent $A$, $s^A$, taking the cut-off type of agent $B$, $s^B$, as given. The equilibria are then obtained by finding intersections of $s^A\left(s^B\right)$ with its inverse.

Start with case $a$ where $s^A \geq s^B$. If agent $A$ has costs $\theta \leq s^B$, he will produce in period 1. Indeed, he knows that, if he does not produce in period 1, the principal will learn that his costs are below $s^B$ since agent $B$ will produce in period 1 and the second-period price $p_2^y$ will satisfy $p_2^y \leq s^B \leq p_1$. Then, agent $A$ finds it optimal to produce in period 1.

The agent with costs $\theta > s^B$ has a rent of $p_1 - \theta$ if he produces in period 1. If he does not produce, he will be offered a price $p_2^n \geq p_1$ since in period 1 the other agent will not have produced either. His rent will be $\delta\left(p_2^n - \theta\right)$. The cut-off type $s^A$ is then

$$s^A = \frac{p_1 - \delta p_2^n}{1-\delta}. \tag{5}$$

The problem of principal $A$ is

$$\begin{cases} \max\limits_{p_1, p_2^n} \Pi_a^{FC} = \left(v_1 - p_1\right)s^A + \delta\left(v_2 - p_2^n\right)\left(p_2^n - s^A\right) \\ \\ \text{s.t.} \quad s^B \leq p_1 \leq p_2^n \leq 1 \text{ and (5).} \end{cases} \tag{6}$$

In case $b$, i.e., $s^A \leq s^B$, by a similar logic, the type which is indifferent between production in period 1 and in period 2 is $s^A \leq s^B$ given by

$$s^A = \frac{p_1 - \delta p_2^y}{1-\delta}. \tag{7}$$

Principal $A$'s problem is

$$
\begin{cases}
\max_{p_1,p_2^y,p_2^n} \Pi_b^{FC} = (v_1 - p_1)\, s^A + \delta \left[ (v_2 - p_2^y) \left(p_2^y - s^A\right) + (v_2 - p_2^n) \left(p_2^n - s^B\right) \right] \\
\\
\text{s.t.} \quad p_1 \le p_2^y \le s^B \le p_2^n \le 1 \text{ and (7).}
\end{cases}
\tag{8}
$$

The next Lemma simplifies further analysis. It says that the principal commits not to change her offer (unless there is evidence that the first-period price was unacceptable for the agent). The reason is that the losses that occur because the agent delays his production outweigh the benefits of an additional production in period 2. This is the same reason as in Sobel and Takahashi (1983) and, more generally, in the bargaining literature where, for example, the seller, selling to a privately informed buyer, restricts sales to the first period.

**Lemma 2** *In case $a$, $p_1 = p_2^n = s^A$. In case $b$, $p_1 = p_2^y = s^A$.*

**Proof.** For case $a$, plug (5) into $\Pi_a^{FC}$ and differentiate it with respect to $p_2^n$ to obtain

$$
\operatorname{sign}\left[\frac{\partial \Pi_a^{FC}}{\partial p_2^n}\right] = \operatorname{sign}\left[2p_1 - 2p_2^n + v_2 - v_1\right].
$$

As $v_1 \ge v_2$, the derivative is negative if $p_2^n > p_1$. Then, $p_2^n = p_1$ and, from (5), $s^A = p_2^n = p_1$.

Case $b$ is proved analogously. ■

Note that in case $b$ the second-period price when no firm produced in period 1, $p_2^n$, is ex post efficient: $p_2^n = \min\left\{\frac{s^B + v_2}{2}, 1\right\}$. The principal commits not to change her offer unless she receives a signal that the first-period offer was too low.

As the principal commits not to increase the (relevant) price in the future, the agent accepts any offer above his costs. Then, the first-period price is also the agent's type which is indifferent about producing in period 1 or 2. The next Lemma characterizes the optimal $s^A$. Figure 2 shows the optimal $s^A$, prices and profits.

**Lemma 3** *In the full-commitment setting, there exists $\theta^{FC} < \frac{v_1}{2}$ such that*

$$
s^A = \left[
\begin{array}{ll}
\frac{v_1}{2}, & \text{if } s^B \le \theta^{FC}, \\
s^B, & \text{if } s^B \in \left[\theta^{FC}, \frac{v_1}{2}\right], \\
\frac{v_1}{2}, & \text{if } s^B \ge \frac{v_1}{2}.
\end{array}
\right.
$$

**Proof.** See Appendix. ■

The optimal cut-off $s^A$ equals $\frac{v_1}{2}$, with the exception of the range $\left[\theta^{FC}, \frac{v_1}{2}\right]$ where it equals $s^B$. When $s^A = \frac{v_1}{2}$ and is above $s^B$, principal $A$ does not use the information. To use the information, the production has to be at most $s^B$. Thus, principal $A$ faces a trade-off: to distort $s^A$ downwards and to use information or not to distort $s^A$ and to ignore information. When $s^B$ becomes close enough to $\frac{v_1}{2}$, the distortion of $s^A$ becomes relatively small and the benefits of information outweigh the losses due to the distortion. This happens at $s^B = \theta^{FC}$. The principal keeps the distortion minimal, that is, $s^A = s^B$. In contrast to the benchmark setting, having $s^A = s^B$ is useful as it allows principal $A$ to detect the strategic rejection of $p_1$. When $s^B$ exceeds $\frac{v_1}{2}$, the principal keeps $s^A$ undistorted at $\frac{v_1}{2}$ and still uses the information.

Let us now turn to the equilibria of the game which are characterized in the next Proposition.

**Proposition 2** *In the full-commitment setting, there is a continuum of symmetric equilibria in which both firms have the same production* $s^A = s^B \in \left[\theta^{FC}, \frac{v_1}{2}\right]$. *There are no other equilibria.*

All the equilibria are symmetric in the full-commitment setting. This means that the two firms always produce simultaneously, whether it be in period 1 or in period 2. The two firms generate the same information (the same partition) and principals use the information from the other firm to discipline their respective agents. In the benchmark setting, there was no need for this and generating the same information was useless. Now, by offering identical contracts, principals can detect strategic rejection of the contract in period 1, that is, rejection when the costs of the agent $\theta$ are below $p_1$, since in this case both agents are supposed to accept the contracts and produce in period 1. Then, principals can contract with their agents in period 2 when no firm produced before offering $p_2^n > p_1$ without a strategic response from the agent in period 1. Indeed, as we see from (7), $p_2^n$ does not affect $s^A$ in case $b$.

These multiple symmetric equilibria can be ranked in terms of their efficiency. As we noted above, a higher first-period production, that is, a higher cut-off $s$, corresponds to a higher and earlier total production and, therefore, a higher efficiency. Then, the "highest" symmetric equilibrium, in which $s^A = s^B = \frac{v_1}{2}$, is the most efficient one. In the autarky, each firm produces at the same level $s = \frac{v_1}{2}$ in period 1; however, principals commit to this take-it-or-leave-it offer and there is no production

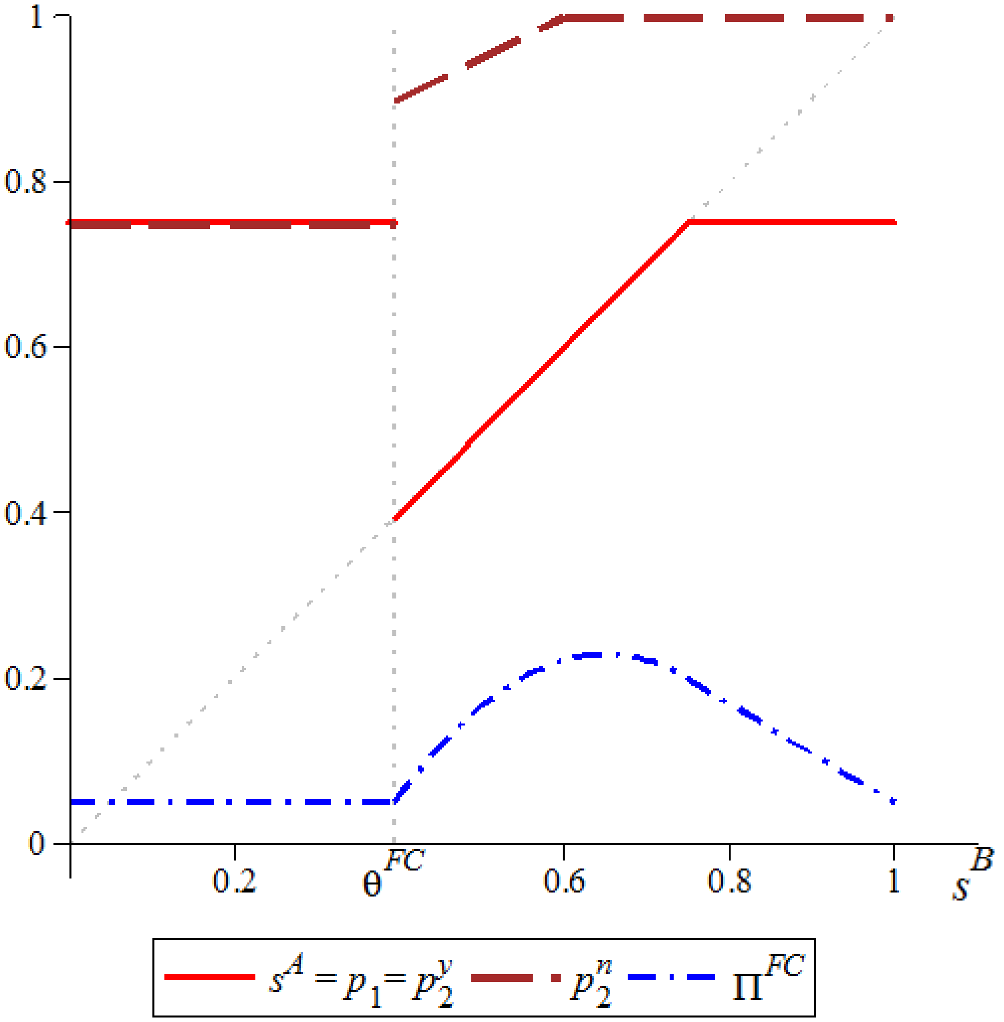


Figure 2: Prices and profits in the full-commitment setting, $v_1 = 1.5$, $v_2 = 1.4$, $\delta = 0.5$. For profits, the scale is adjusted.

in period 2. Information obtained from the offer allows principals to offer a higher price $p_2^n$ in period 2 without a possibility of strategic rejection of their offer in period 1. This is the strategic effect which increases efficiency. In the highest symmetric equilibrium informational externalities only improve contracting in period 2, as compared to the autarky. In other equilibria, they also cause a higher delay in period 1 which is a standard negative effect. Comparison with the autarky becomes then ambiguous.

A more efficient equilibrium is always associated with higher prices offered to the agents which means higher rents for them. The profit-maximizing equilibrium is, however, an intermediate one as we can see in Figure 2. Then, in equilibria below the profit-maximizing one the principals, the agents and consumers all want to move to a higher equilibrium. In equilibria above the profit-maximizing one, the preferences of the principals are opposed to those of agents and consumers.

# 3 Competition

In this Section, we introduce competition. We allow for a great generality parametrizing possible effects of competition in the following way. *Intra-period competition* arises when the two firms produce in the same period.[18] If this happens in period 1, each principal earns $\beta_1 v_1$, $\beta_1 \leq 1$, and if in period 2, each principal earns $\beta_2 v_2$, $\beta_2 \leq 1$. *Inter-period competition* arises when the two firms produce in two different periods. Under competition from past sales, a firm suffers when it is the second to produce and in this case its revenues decrease to $\gamma_p v_2$, $\gamma_p \leq 1$. The first firm then has a first-mover advantage. Under competition from future sales, the firm which first produces suffers from the market's anticipation that the other firm will produce later. Its revenues then decrease to $\gamma_f v_1$, $\gamma_f \leq 1$.[19] We keep our assumption that it is always efficient to produce, that is, $\beta_1 v_1$, $\beta_2 v_2$, $\gamma_p v_2$ and $\gamma_f v_1$ are all greater than 1. A stronger competition means that revenues fall more and, therefore, it corresponds to a lower $\beta_1$, etc.[20]

[18] A good interpretation is that agents are R&D divisions and the principals are headquarters. The agents develop a new product that is then produced by principals at zero cost and sold in the market.

[19] More precisely, its revenues become a combination of $\gamma_f v_1$ and $v_1$ weighted by the probability that the competitor will introduce the product in period 2.

[20] Note that competition affects the uninformed party. It can also affect the informed party, for example, when suppliers (with known costs) sell the input to downstream firms with unknown valuations. The effects of competition would still be similar.

There are four possible effects of competition on the optimal cut-off type $s^A$: competition may increase or decrease $s^A$ in case $a$ (when $s^A$ is above $s^B$) or in case $b$ (when $s^A$ is below $s^B$). It turns out that there is one-to-one correspondence between the four kinds of competition described above and these four effects. This is described in Table 1 below. We say that the set of symmetric equilibria *shrinks* if the interval of the values of $s^B$ such that $s^A = s^B$ decreases. In some cases, this interval may even disappear and then there will be two asymmetric equilibria. When there are already no symmetric equilibria, the asymmetric ones become more asymmetric in the sense that the ratio $\frac{s^A}{s^B}$ goes further away from one. We also say that the set of symmetric equilibria *expands* to describe the opposite phenomenon.

Table 1: Effects of different kinds of competition

| *Type of competition* | *Setting affected* | *Cut-off type s* | *Set of sym. eq.* |
|---|---|---|---|
| Intra-period 1, $\beta_1 \downarrow$ | both $BM$ and $FC$ | decreases in case $b$ | shrinks |
| Intra-period 2, $\beta_2 \downarrow$ | both $BM$ and $FC$ | increases in case $a$[21] | shrinks |
| Past sales, $\gamma_p \downarrow$ | only $BM$ | increases in case $b$ | expands |
| Future sales, $\gamma_f \downarrow$ | only $BM$ | decreases in case $a$ | expands |

Table 1 should be interpreted in the following way. Each line describes the effects of one of the four kinds of competition. Consider, for example, the first line which corresponds to a stronger intra-period competition in period 1, i.e., a lower $\beta_1$. It affects both the benchmark and full-commitment settings. It decreases the optimal cut-off type in case $b$ (and does not affect it in case $a$). The set of symmetric equilibria shrinks.

Below, we start with inter-period competition from past sales and study it in some detail. In particular, we characterize optimal $s^A$ in the benchmark setting and show that symmetric equilibria might appear. We summarize the results for the three other kinds of competition, providing intuition for their effects. Then, we provide an example of how a standard model of competition (Hotelling's spatial model) translates into intra- and inter-period competition.

Finally, we show that, independently of the strength and the form of competition, symmetric equilibria exist for a larger range of parameters in the full-commitment

[21] Only in the benchmark setting.

setting than in the benchmark setting.

## 3.1 Inter-period competition from past sales

A stronger inter-period competition from past sales, or a bigger first-mover advantage, corresponds to a lower $\gamma_p$. It hurts firm $A$ when firm $A$ produces in period 2 while firm $B$ has produced in period 1, that is, when $s^A < p_2^y \leq s^B$. Then, in case $a$, competition from past sales does not have any effect.

Consider case $b$ and let us start with the benchmark setting. Problem (3) should be replaced by

$$\begin{cases} \max\limits_{p_1, p_2^y, p_2^n} \Pi_b^{BM} = (v_1 - p_1) p_1 + \delta \left[ \left( \gamma_p v_2 - p_2^y \right) \left( p_2^y - p_1 \right) + \left( v_2 - p_2^n \right) \left( p_2^n - s^B \right) \right] \\ \\ \text{s.t.} \quad p_1 \leq p_2^y \leq s^B \leq p_2^n \leq 1. \end{cases}$$

The only difference with (3) is that, when production takes place after that of the other firm, the revenues are multiplied by $\gamma_p$. As intuition suggests, a stronger competition (lower $\gamma_p$) decreases profits (apply envelope theorem). The prices become (compare with (4))

$$\begin{aligned} p_1 &= \min \left\{ \frac{v_1 - \delta \gamma_p v_2 + \delta p_2^y}{2}, s^B \right\}, \\ p_2^y &= \min \left\{ \frac{\gamma_p v_2 + p_1}{2}, s^B \right\}, \\ p_2^n &= \min \left\{ \frac{v_2 + s^B}{2}, 1 \right\}. \end{aligned}$$

We see that $p_1 = s^A$ decreases with $\gamma_p$ and, therefore, a stronger competition from past sales (lower $\gamma_p$) will increase $p_1$. A lower $\gamma_p$ makes waiting until period 2 less attractive and therefore increases production in period 1.

Lemma 10 in the Appendix characterizes the optimal $p_1$. In particular, compared to Lemma 1, a new case 0 appears: whenever $\frac{v_1 + (2-\delta) v_2}{2 \gamma_p v_2 + 2 - \delta} \geq 1$, $s^A = s^B$ in some range. This obviously gives rise to symmetric equilibria that we did not observe before, in Section 2. See Figure 3.

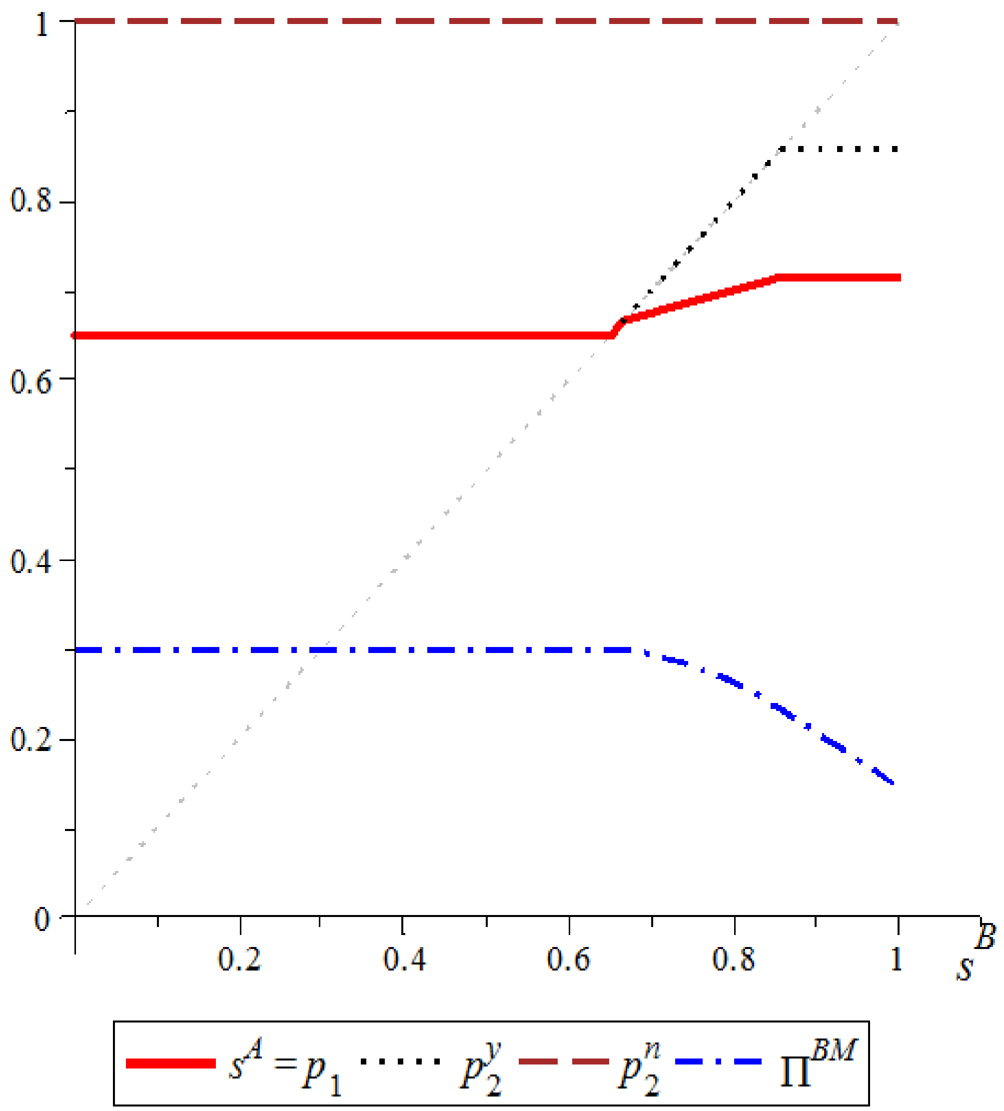


Figure 3: Prices and profits in the benchmark setting (case 0) under inter-period competition from past sales, $v_1 = 1.5$, $v_2 = 1.4$, $\delta = 0.8$, $\gamma_p = \frac{5}{7}$. For profits, the scale is adjusted.

The next Lemma characterizes equilibria in the benchmark setting.

**Lemma 4** *In the benchmark setting, under inter-period competition from past sales, there are the following equilibria:*

0. *If* $\frac{v_1+(2-\delta)v_2}{2\gamma_p v_2+2-\delta} \geq 1$*, there is a continuum of symmetric equilibria in which both firms produce at the same level* $s^A = s^B \in \left[\frac{v_1-\delta v_2+\delta}{2}, \frac{v_1-\delta\gamma_p v_2}{2-\delta}\right]$*. There are no asymmetric equilibria.*

1,2. *If* $\frac{v_1+(2-\delta)v_2}{2\gamma_p v_2+2-\delta} < 1$*, there are only two equilibria and they are asymmetric.*

**Proof.** Analogous to the proof of Proposition 1. ■

Thus, under competition from past sales, symmetric equilibria appear whenever $\frac{v_1+(2-\delta)v_2}{2\gamma_p v_2+2-\delta} \geq 1$. A lower $\gamma_p$ makes this condition easier to satisfy and it expands the interval $\left[\frac{v_1-\delta v_2+\delta}{2}, \frac{v_1-\delta\gamma_p v_2}{2-\delta}\right]$, where $s^A = s^B$, to the right. In the same spirit, a lower $\gamma_p$ makes the asymmetric equilibria of cases 1 and 2 of the benchmark setting less asymmetric, in the sense that the cut-offs in the two firms in period 1 become closer to each other. Indeed, in these equilibria in one of the firms (say, $A$) the production is higher than that of firm $B$, $s^A > s^B$. Then, $s^A$ is not affected by $\gamma_p$, while $s^B$ decreases with $\gamma_p$. The ratio of the higher cut-off to the smaller one (from Lemma 10 in Appendix, case 1)

$$\frac{s^A}{s^B} = 2\frac{v_1 - \delta v_2 + \delta}{(2+\delta)\, v_1 - \delta v_2 \left(2\gamma_p + \delta\right) + \delta^2}.$$

increases in $\gamma_p$ and, therefore, a lower $\gamma_p$ moves it towards one. The same holds for case 2.

Consider now the full-commitment setting. There, the principals commit not to contract with the agent in period 2 (unless the costs are revealed to be higher than the first-period price), and so when $s^A \leq s^B$, $s^A = p_1 = p_2^y$ (Lemma 2). Therefore, in the full-commitment setting, inter-period competition from past sales does not have any effect.

The next Proposition summarizes the main effects of the competition from past sales.

**Proposition 3** *A stronger inter-period competition from past sales (lower* $\gamma_p$*) increases the cut-off type* $s$ *in case b and expands the range of symmetric equilibria to the right in the benchmark setting. It does not affect the full-commitment setting.*

## 3.2 Three other kinds of competition

### 3.2.1 Intra-period competition in period 1

Consider now intra-period competition in period 1. If both firms produce in period 1, they obtain only $\beta_1 v_1$ each, $\beta_1 \leq 1$. A stronger intra-period competition corresponds to a lower $\beta_1$. It affects both the benchmark and the full-commitment settings.

In case $a$, the principals' profits in period 1, instead of $(v_1 - p_1)\, s^A$, become $(\beta_1 v_1 - p_1)\, s^B + (v_1 - p_1)\left(s^A - s^B\right)$, as if she was selling $s^A$ units, and the first $s^B$ of them at the price $\beta_1 v_1$ and the other $s^A - s^B$ at the price $v_1$. The profits decrease when $\beta_1$ becomes smaller; however, the marginal profits are unaffected and, therefore, the optimal prices do not depend on $\beta_1$.

In case $b$, in period 1 the principal obtains $(\beta_1 v_1 - p_1)\, s^A$ and therefore all prices and the cut-off type $s^A$ should be adjusted downwards for $v_1$ becoming effectively $\beta_1 v_1$.

Consider the set of symmetric equilibria in the full-commitment setting. The lower end of the interval where $s^A = s^B$ is defined as the value of $s^B$, such that the autarky profits, i.e., when $s^A > s^B$, equal profits when $s^A = s^B$. From the discussion above, it is clear that $\beta_1$ has an identical effect on both profits and, therefore, the lower end of the interval where $s^A = s^B$ does not depend on $\beta_1$. In the benchmark setting, the lower end of the interval $s^A = s^B$ is the autarky production that does not depend on $\beta_1$.[22] The upper end of the interval where $s^A = s^B$ is the optimal $s^A$ when the constraint $s^A \leq s^B$ stops binding. It decreases since a lower $\beta_1$ decreases the optimal $s^A$. In both settings, the set of symmetric equilibria thus shrinks from the right.

Symmetric equilibria may even cease to exist. Take, for example, the full-commitment setting. There, the interval where $s^A = s^B$ becomes $\left[\theta^{FC}, \frac{\beta_1 v_1}{2}\right]$.[23] Then, if $\frac{\beta_1 v_1}{2}$ is smaller than $\theta^{FC}$, there are no symmetric equilibria. Instead, two asymmetric equilibria appear in which in one firm the cut-off $s$ equals $\frac{v_1}{2}$ and in the other $s = \frac{\beta_1 v_1}{2}$. The same may happen in case 0 of the benchmark setting.[24] Whenever symmetric equilibria do not exist, asymmetric equilibria arise with the

[22] We implicitly assume that there is also competition from past sales (or future sales) under which symmetric equilibria in the benchmark setting may exist.

[23] Once $\beta_1$ is introduced, $s^A$ becomes equal to $\frac{\beta_1 v_1}{2}$ in case $b$.

[24] In particular, the condition that separates case 0 from cases 1 and 2 in the benchmark setting becomes

same structure, as in cases 1 and 2 of the benchmark setting.

The next Proposition summarizes the effects of a lower $\beta_1$.

**Proposition 4** *A stronger intra-period competition in period* 1 *(lower* $\beta_1$*) decreases the cut-off type* $s$ *in case* $b$ *and shrinks the range of symmetric equilibria from the right in both the benchmark and the full-commitment settings.*

### 3.2.2 Intra-period competition in period 2

Suppose now that there is intra-period competition in period 2. If both firms produce in period 2, they obtain only $\beta_2 v_2$ each, $\beta_2 \leq 1$. A stronger intra-period competition corresponds to a lower $\beta_2$.

The crucial difference (and complication) of this kind of competition is that the profits of each firm are now affected not only by the other agent's cut-off type $s$ (in period 1), but also by his cut-off type in period 2 in the case where nobody produced in period 1 which is $p_2^n$. In other words, the strategy of each firm is now a two-dimensional vector $(s, p_2^n)$.

In the benchmark setting, intra-period competition in period 2 hurts profits in both cases $a$ and $b$ since in each there is some probability that none of the agents produces in period 1. In case $a$, exposure to this kind of competition occurs whenever $\theta \in (s^A, \min\left\{p_2^{n,A}, p_2^{n,B}\right\}]$. Then, the desire to escape from this competition will make principal $A$ go for more production in period 1, that is, increase $s^A$. This implies the set of symmetric equilibria shrinks from the left. In case $b$, exposure to intra-period competition in period 2 occurs whenever $\theta \in (s^B, \min\left\{p_2^{n,A}, p_2^{n,B}\right\}]$. It does not depend on $s^A$ and, therefore, the optimal $s^A$ does not change. This implies that the upper end of the interval where $s^A = s^B$ is not affected. Symmetric equilibria may disappear in the benchmark setting since the interval $(\frac{v_1 - \delta v_2 + \delta}{2}, \frac{v_1 - \delta \gamma v_2}{2-\delta}]$ may not exist once $v_2$ is decreased to $\beta_2 v_2$ at its lower end.

In the full-commitment setting, only case $b$ is affected since in case $a$ there is no production in period 2. In case $b$, the profits are not affected at the margin and,

$$\frac{v_1 + (2-\delta)\, v_2}{\frac{2v_1}{\delta}(1-\beta_1) + 2\gamma_p v_2 + 2 - \delta} \geq 1.$$

(when $\beta_1 = 1$, we obtain the condition in Lemma 4).

Its left-hand side increases with $\beta_1$; therefore, a stronger competition (lower $\beta_1$) makes it more difficult to satisfy.

therefore, $s^A$ still equals $\frac{v_1}{2}$ unless constrained by $s^B$. Then, $\theta^{FC} < \frac{v_1}{2}$ still exists since, even at the lowest possible value of $\beta_2 = \frac{1}{v_2}$, principal $A$ prefers to distort $s^A$ downwards to $s^B$ to be able to use the information if $s^B$ is sufficiently close to $\frac{v_1}{2}$ (from below). Therefore, symmetric equilibria always exist in the full commitment setting.

The next Proposition summarizes the effects of a stronger intra-period competition in period 2.

**Proposition 5** *A stronger intra-period competition in period* 2 *(lower* $\beta_2$*) shrinks the range of symmetric equilibria from the left in both settings and increases the cut-off type* $s$ *in case* $a$ *of the benchmark setting.*[25]

### 3.2.3 Inter-period competition from future sales

Finally, the revenues in period 1 may be hurt if the competitor is expected to sell in period 2. Suppose that firm $A$ earns only $\gamma_f v_1$, $\gamma_f \leq 1$, in period 1 if firm $B$ sells in period 2.

Under this kind of competition, as well as under intra-period competition in period 2, the profits of the principals are affected not only by the other agent's cut-off type $s$ (in period 1) but also by his cut-off type in period 2 in the case where nobody produced in period 1 which is $p_2^n$. In other words, the strategy of each firm is a two-dimensional vector $(s, p_2^n)$.

Case $b$ in each setting is not affected as, there, firm $A$ produces only if firm $B$ does. In case $a$, when only firm $A$ is to sell in period 1 (that is, when the costs of the agents are between $s^B$ and $s^A$), the market knows that for types of the agent lower than $p_2^{y,B}$, firm $B$ will sell in period 2. We obtain immediately that the full-commitment setting is not affected at all since there $s^A = p_2^y$, that is, there is no production in period 2 by firm $B$ if firm $A$ produces in period 1.

In the benchmark setting, the profits in case $a$ decrease. The effect on marginal profits, and therefore prices and $s^A$, depends on whether the constraint $p_2^{y,B} \leq s^A$ binds. If it binds, $p_2^{y,B} = s^A$, the market knows that the other firm will sell in period 2 and therefore, principal $A$ gets only $\gamma_f v_1$. Then, when computing $s^A$, we need to replace $v_1$ by $\gamma_f v_1$ in the expressions we had before. If it does not bind, $p_2^{y,B} = \frac{v_2+s^B}{2}$,

[25] In the full-commitment setting, symmetric equilibria never disappear.

firm $B$ sells in period 2 if the cost of the agent is between $s^B$ and $p_2^{y,B}$; it does not sell in period 2 if the cost of the agent is between $p_2^{y,B}$ and $s^A$. Then, assume that firm $A$ earns a weighted sum of revenues with and without competition from future sales, that is, $\frac{v_1}{s^A - s^B}\left[\left(p_2^{y,B} - s^B\right)\gamma_f + s^A - p_2^{y,B}\right]$. Future sales still reduce marginal profits and production in period 1, though to a lesser extent than when $p_2^{y,B} = s^A$.

The next Proposition summarizes the effects of competition from future sales.

**Proposition 6** *A stronger competition from future sales (lower $\gamma_f$) expands the range of symmetric equilibria to the left and decreases the cut-off type $s$ in case a in the benchmark setting. It does not affect the full-commitment setting.*

### 3.3 Example: Hotelling's competition

We show here how Hotelling's model of horizontal differentiation translates into the intra- and inter-period competition described above. Consider a "linear city" of length 1 populated by consumers distributed uniformly across the city. Firm $A$ is located at the left end of the city, while the firm $B$ is located at the right end and consumers have transportation cost $c < \frac{2}{3}$ per unit of length. The (per period) valuation of each consumer is 1. The product is non-durable and, therefore, if a firm produces in period 1, it will sell it for two periods.[26] Then, if a consumer located at $x$ buys the product from firm $A$ at price $p$, his utility is

$$u = 1 - p - xc.$$

If firm $A$ is alone selling the product in the market and sets its price equal to $p$, it sells to consumers located below $\frac{1-p}{c}$. The price that maximizes its profits $p\frac{1-p}{c}$ is $p = \frac{1}{2}$ and the resulting profits are $\frac{1}{4c}$.[27] Then,

$$v_1 = \frac{1}{4c}\left(1 + \delta\right) \text{ and } v_2 = \frac{1}{4c}.$$

When the two firms compete offering $p_A$ and $p_B$, the consumer located at $x^*$ such that $p_A + x^*c = p_B + (1 - x^*)\,c$ is indifferent between buying from firm $A$ and firm

[26] "Production" by the agent can be thought as the development of a new product and once the product has been developed, the production costs are zero.

[27] If firms could choose their location, the monopolist would place itself in the middle of the interval and earn $1 - \frac{c}{2}$. All the qualitative results would still hold.

$B$. Firm $A$ maximizes $p_A x^* = p_A \frac{p_B - p_A + c}{2c}$ taking $p_B$ as given. In the equilibrium, $p_A = p_B = c$ and each firm earns $\frac{c}{2}$. Then,

$$\begin{aligned}
\beta_1 v_1 &= \frac{c}{2}(1+\delta), \\
\beta_2 v_2 &= \frac{c}{2}, \\
\gamma_p v_2 &= \frac{c}{2}, \\
\gamma_f v_1 &= \frac{1}{4c} + \delta \frac{c}{2}.
\end{aligned}$$

We can now find $\beta_1$ and other parameters:

$$\beta_1 = \beta_2 = \gamma_p = 2c^2 \text{ and } \gamma_f = \frac{1 + 2c^2\delta}{1+\delta} > 2c^2.$$

Lower transportation cost $c$ increases profits when the firm is a monopolist, $v_1$ and $v_2$, since more consumers are served. In the competition case, lower $c$ decreases profits since competition becomes more intense. Both effects lead to stronger intra- and inter-period competition, that is, lower $\beta_1$ and other parameters.

We can study the effect of a lower $c$ in two steps. At the first step consider $\beta_1 = \beta_2 = \gamma_p = \gamma_f$ and increase $\gamma_f$ at the second one. Suppose that $v_2$ is high enough so all the types produce after two periods, that is, $p_2^n = 1$. The first step is then equivalent to a proportional reduction in $v_1$ and $v_2$ and our previous characterization applies (Propositions 1 and 2). At the second step, inter-period competition from future sales is relaxed ($\gamma_f$ increases). The equilibria in the benchmark setting become then even more asymmetric since the leader increases its first-period production while the follower does not change it (Proposition 6). The full-commitment setting is not affected since the two firms never produce sequentially.

## 3.4 Comparison of the equilibria in the benchmark and full-commitment settings

When competition is absent, there are only asymmetric equilibria in the benchmark setting (Proposition 1) and only symmetric ones in the full-commitment setting (Proposition 2). Then, trivially, the set of symmetric equilibria in the benchmark setting is contained in the set of symmetric equilibria in the full-commitment setting. Under competition, symmetric equilibria may appear in the benchmark setting (Lemma 4) or disappear in the full-commitment setting (Section 3.2.1). However, as we show below in Proposition 7, the sets of symmetric equilibria are still nested.

In order to simplify the exposition, we first prove Lemma 5. In its statement, the optimal $s^A$ is meant.

**Lemma 5** *(i) If* $s^A = s^B$ *in the benchmark setting, then* $s^A = s^B$ *in the full-commitment setting and the profits are the same in the two settings.*

*(ii) If* $s^A \neq s^B$ *(and* $s^A < 1$*) in either of the two settings, then profits in the benchmark setting are strictly higher than in the full-commitment setting.*

**Proof.** See Appendix. ■

The next Proposition presents the main result of this section.

**Proposition 7** *The set of symmetric equilibria in the benchmark setting is contained in the set of symmetric equilibria in the full-commitment setting. Moreover, the inclusion is strict unless the set of symmetric equilibria in the full-commitment setting is empty.*

Once Lemma 5 is proven, the proof of Proposition 7 is simple and therefore omitted. Indeed, from Lemma 5(i) it immediately follows that any symmetric equilibrium in the benchmark setting is also a symmetric equilibrium in the full-commitment setting. The fact that the inclusion is strict follows from Lemma 5(ii). The intuition is the following. In the full-commitment setting, principals commit not to change the price in period 2 unless they learn from the other firm that the first-period offer was too low. As soon as $s^A \neq s^B$, this commitment is costly since types above $s^A$ (in case $a$) or between $s^A$ and $s^B$ (in case $b$) are shut down. In the benchmark setting, contracting in period 2 is always ex post optimal. Hence, when the agents are strategic, the principals have an additional motive to set $s^A = s^B$.

Another way to state Proposition 7 is that the range of parameters for which there exists a symmetric (asymmetric) equilibrium expands (shrinks) when the setting is changed from the benchmark to the full commitment.

# 4 No commitment

In this Section, we analyze the no-commitment setting in which the principals cannot commit to two-period contracts. To simplify the exposition, we assume away competition as we did in Section 2. We first derive the best response function $s^A\left(s^B\right)$ and then find the equilibria. After that we prove that the set of symmetric equilibria is larger in the no-commitment setting than in the full-commitment one. In other words, symmetric behavior of the two firms is more likely the more severe are the contractual problems.

At the beginning of period 1 principal $A$ offers a first-period price $p_1$.[28] If the agent rejects the offer, she offers $p_2^y$ or $p_2^n$ depending on whether the other firm produced in period 1 or not. These second-period prices are ex post optimal and higher than $p_1$.

As in the full-commitment setting, the agents are strategic, that is, they accept the first-period offer only when it brings a higher rent than the expected rent in period 2. The cut-off type of the agent which is indifferent as to whether he produces in period 1 or in period 2 is $s^A$ given, as in the full commitment setting, by (5) and (7), respectively, depending on whether $s^A$ is higher or lower than $s^B$. Now, however, second-period prices $p_2^y$ and $p_2^n$ are not announced in period 1 but expected to arise as ex post optimal in period 2.

The game is solved backwards. In period 2, principal $A$ chooses the optimal $p_2^n$ and $p_2^y$ knowing that she faces an agent with costs higher than $s^A$. In period 1, when deciding about $p_1$ and, thus, about $s^A$, she takes into account that $p_1$ will affect second-period prices. Principal $A$ also takes the behavior of firm $B$, summarized by the cut-off type $s^B$, as given.

As in the benchmark and full-commitment settings, there are two cases to consider. Let us start with case $a$, $s^A \geq s^B$.

If $p_1$ is rejected by agent $A$, the fact that firm $B$ has not produced either gives principal $A$ no information as $s^A \geq s^B$. Thus, in period 2 she maximizes $\left(v_2 - p_2^n\right)\left(p_2^n - s^A\right)$ which gives

$$p_2^n = \min\left\{\frac{v_2 + s^A}{2}, 1\right\}. \tag{9}$$

[28]Under no commitment, principals cannot commit to future positive payments. The credit constraint assumed in Section 2.3 can then be relaxed.

Then, the problem of principal $A$ in period 1 is

$$\begin{cases} \max\limits_{p_1} \Pi_a^{NC} = (v_1 - p_1)\, s^A + \delta\,(v_2 - p_2^n)\left(p_2^n - s^A\right) \\ \\ \text{s.t.} \quad s^B \leq p_1 \leq 1,\ (5) \text{ and } (9). \end{cases} \tag{10}$$

When $s^A \leq s^B$ (case $b$), the principal's problem in period 1 is

$$\begin{cases} \max\limits_{p_1} \Pi_b^{NC} = (v_1 - p_1)\, s^A + \delta\left[(v_2 - p_2^y)\left(p_2^y - s^A\right) + (v_2 - p_2^n)\left(p_2^n - s^B\right)\right] \\ \\ \text{s.t.} \;\; p_1 \leq s^B,\ (7),\ p_2^y = \min\left\{\frac{v_2 + s^A}{2}, s^B\right\} \text{ and } p_2^n = \min\left\{\frac{v_2+s^B}{2}, 1\right\}. \end{cases} \tag{11}$$

The next Lemma characterizes the optimal $s^A$. For a numerical example see Figure 4.

**Lemma 6** *In the no-commitment setting, there exist* $\theta_1^{NC} < \frac{v_1}{2}$*,* $\theta_2^{NC} > \frac{v_1}{2}$ *and* $\widetilde{v}_2 \in \left(\frac{v_1-\delta v_2}{2-\frac{3}{2}\delta}, \frac{v_1-\delta v_2}{2(1-\delta)}\right)$ *such that*

$$s^A = \begin{bmatrix} \begin{bmatrix} \min\left\{\frac{v_1-\delta v_2}{2(1-\delta)}, 1\right\}, & \text{if } 2 - v_2 \leq \widetilde{v}_2 \\ \min\left\{\frac{v_1-\delta v_2}{2-\frac{3}{2}\delta}, 1\right\}, & \text{if } 2 - v_2 > \widetilde{v}_2 \end{bmatrix} & \text{if } s^B \leq \theta_1^{NC}, \\ s^B, & \text{if } s^B \in \left[\theta_1^{NC}, \min\left\{\frac{v_1-\delta v_2}{2(1-\delta)}, \theta_2^{NC}\right\}\right], \\ \min\left\{\frac{v_1-\delta v_2}{2(1-\delta)}, 1\right\}, & \text{if } s^B \in \left[\frac{v_1-\delta v_2}{2(1-\delta)}, \theta_2^{NC}\right] \text{ (when this region exists)}, \\ \min\left\{\frac{v_1-\delta v_2}{2-\frac{3}{2}\delta}, 1\right\}, & \text{if } s^B \geq \theta_2^{NC}. \end{bmatrix}$$

**Proof.** See Appendix. ■

The behavior of the optimal response function $s^A$ is similar to the full-commitment case. When $s^B$ is small, $s^A$ is higher and, therefore, the information coming from firm $B$ is not used. Then, at $s^B = \theta_1^{NC}$, $s^A$ falls and becomes equal to $s^B$ as, in this way, principal $A$ uses the information but keeps the distortion to a minimum. For some range of $s^B$, $s^A$ is equal to $s^B$. When $s^B$ becomes close to one, there are three possibilities. First, $s^A$ may continue to be equal to $s^B$ for $s^B$ up to one. Second, $s^A$ may become constant, as in the full-commitment setting. Third, $s^A$ may become constant and then fall down again at $s^B = \theta_2^{NC}$ to another level. The second jump at $s^B = \theta_2^{NC}$ is unique to the no-commitment setting.

The second case is illustrated in Figure 4. When $s^A$ becomes lower than $s^B$, an increase in $s^B$ causes principal $A$ to offer a higher $p_2^y$; however, $s^A$ stays constant. That is, principal $A$ suffers more and more from the inability to commit not to increase the second-period price.

Finding equilibria is again straightforward as there are only symmetric equilibria.

**Proposition 8** *In the no-commitment setting, there is a continuum of symmetric equilibria in which both principals set the same first-period price $p_1$ that results in the same production $s^A = s^B \in \left[\theta_1^{NC}, \min\left\{\frac{v_1-\delta v_2}{2(1-\delta)}, \theta_2^{NC}\right\}\right]$. There are no other equilibria.*

As in the full-commitment setting, there are multiple symmetric equilibria and no asymmetric ones. The two firms always produce together, either in period 1 or in period 2. They generate the same partition of the type space, i.e., the same information, and the principals use it to commit not to improve their offer, $p_2^y = p_1 = s^B$. The most efficient equilibrium is the "highest" symmetric equilibrium, that is, the one in which $s^A = s^B = \min\left\{\frac{v_1-\delta v_2}{2(1-\delta)}, \theta_2^{NC}\right\}$. The comparison with the autarky is ambiguous for two reasons. First, unlike the full-commitment setting, the autarky is relatively efficient since principals cannot shut down high cost types in period 2. Second, even in the highest equilibrium, first-period production may be greater or smaller than in the autarky.[29]

We now show that the set of symmetric equilibria in the no-commitment setting is larger than the one in the full-commitment setting. For this, we use Lemma 7 which is similar to Lemma 5 and has a similar proof (omitted).

**Lemma 7** *(i) If $s^A = s^B$ in the full-commitment setting, then $s^A = s^B$ in the no-commitment setting as well and the profits are the same in the two settings.*

*(ii) If $s^A \neq s^B$ (and $s^A < 1$) in either of the two settings, then profits in the full-commitment setting are strictly higher than in the no-commitment setting.*

The proof is based on the fact that $\Pi^{FC} \geq \Pi^{NC}$ since the no-commitment solution is feasible in the full-commitment setting. The equality is reached if and only if all the prices are the same.

[29] As in the full-commitment setting, a more efficient equilibrium is always associated with higher prices offered to the agents which means higher rents for them. The profit-maximizing equilibrium is, however, an intermediate one as we can see in Figure 4.

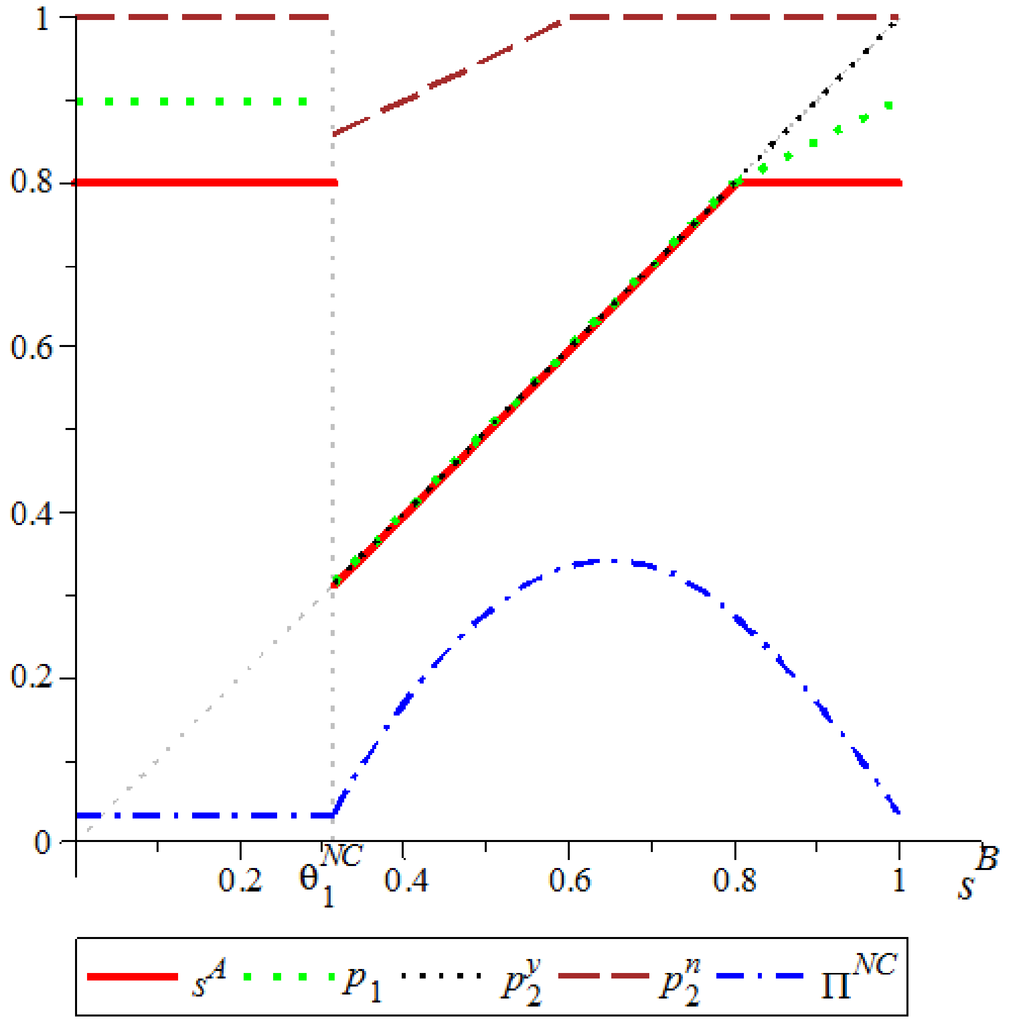


Figure 4: Prices and profits in the no-commitment setting, $v_1 = 1.5$, $v_2 = 1.4$, $\delta = 0.5$. For profits, the scale is adjusted.

**Proposition 9** *The set of symmetric equilibria in the full-commitment setting is contained in the set of symmetric equilibria in the no-commitment setting. Moreover, the inclusion is strict unless the set of symmetric equilibria in the no-commitment setting is empty.*

From Lemma 7(i) it immediately follows that any symmetric equilibrium in the full-commitment setting is also a symmetric equilibrium in the no-commitment setting. The fact that the inclusion is strict follows from Lemma 7(ii). Intuitively, in the no-commitment setting, when $p_1$ is smaller than $s^B$, the principals cannot commit not to offer $p_2^y > p_1$. However, if $p_1$ is equal to $s^B$, it effectively commits the principals not to increase $p_2^y$ as then $p_1 = p_2^y = s^B$. Thus, as compared to the full-commitment setting, there is an additional incentive for the principals to set $s^A = s^B$.

# 5 Alternative information structures

In this Section, we consider two alternative information structures. First, we study the setup where the costs of the agents are independent with some probability, while still the same with a complementary probability. Second, we assume that, after production by one firm in period 1, the other can learn the costs exactly through reverse engineering. For ease of analysis and exposition, we assume, as in Section 2, that there is no competition.

## 5.1 Imperfect correlation of costs

Here, we relax the assumption that the costs of the two agents are exactly the same. Suppose that there are two available technologies and in period 1 neither the principals nor the agents know if they use the same technology as the other firm. They only know that it is the same with probability $\lambda$. After period 1 they learn not only about the production of the other firm but also which technology it uses. Then, with probability $\lambda$ the costs of the two agents are the same, while with probability $1 - \lambda$ they are independent. In the latter case, the production of the other firm in period 1 does not reveal any information. A higher $\lambda$ is interpreted as more, or better, information being revealed by each firm in expectation.

When $s^A \geq s^B$ (case $a$), firm $A$ does not use information it receives from firm $B$. Therefore, case $a$ is not affected by $\lambda$.

Consider case $b$ of the benchmark setting. Denote $p_2^{y,c}$ and $p_2^{n,c}$ the prices, conditional on the production of the other firm in period 1, offered by the the principals in period 2, when the costs of the two agents turn out to be the same, and $p_2^i$ the price offered by the principals in period 2, when the costs of the two agents are independent. Problem (3) becomes

$$\begin{cases} \max_{p_1, p_2^{y,c}, p_2^{n,c}, p_2^i} \Pi_b^{BM} = (v_1 - p_1)\, p_1 \\ +\delta \begin{bmatrix} \lambda \left( (v_2 - p_2^{y,c}) \left(p_2^{y,c} - p_1\right) + \left(v_2 - p_2^{n,c}\right) \left(p_2^{n,c} - s^B\right) \right) \\ + (1-\lambda)\left(v_2 - p_2^i\right)\left(p_2^i - p_1\right) \end{bmatrix} \\ \\ \text{s.t. } p_1 \leq p_2^{y,c} \leq s^B \leq p_2^{n,c} \leq 1, \; p_2^i \in [p_1, 1]\,. \end{cases} \tag{12}$$

It is intuitive, and can be easily shown, that $p_1 = s^A$ decreases with $\lambda$ since more information makes waiting more profitable.[30] This implies that the set of symmetric equilibria shrinks from the right.

In the full-commitment setting, case $b$, the cut-off type $s^A$, instead of (7), becomes

$$s^A = \frac{p_1 - \delta\left[\lambda p_2^{y,c} + (1-\lambda)\, p_2^i\right]}{1-\delta}. \tag{13}$$

When $\lambda = 1$, principal $A$ commits not to increase the price in period 2 (Lemma 2). For $\lambda < 1$, she has even fewer reasons to contract in period 2 and Lemma 2 extends to this case as well. That is, $p_1 = p_2^{y,c} = p_2^i = s^A$ and we obtain the same solution as the one characterized in Lemma 3. Then, when $p_1 = s^B$, profits increase with $\lambda$ since a higher $\lambda$ increases the chances that the costs of the agent are known to be higher than $s^B$, in which case some production takes place in period 2. In case $a$ the profits are unaffected by $\lambda$ and, therefore, $\theta^{FC}$ increases with a lower $\lambda$. When $\lambda = 0$, no information is ever revealed by the other firm and $p_1 = \frac{v_1}{2}$ for any $s^B$, i.e., $\theta^{FC} = \frac{v_1}{2}$. This is the autarky solution.

In the no-commitment setting, the cut-off type $s^A$ is also given by (13). Since there is no commitment to two-period contracts, at least $p_2^i$ will be higher than $p_1$ and, thus, $s^A$ will be lower than $p_1$. There is now a strategic effect of more

[30] If $p_2^i = p_2^{y,c} = \frac{v_2+p_1}{2} < s^B$, then $p_1$ does not change with $\lambda$. Indeed, (12) becomes (3) that does not depend on $\lambda$.

information:

$$\frac{\partial s^A}{\partial \lambda} = \frac{\delta}{1-\delta}\left(p_2^i - p_2^{y,c}\right) > 0.$$

This is the precise sense in which the agent becomes softer in bargaining in period 1 when more information is anticipated in the future. This effect calls for more production in period 1. The strategic effect is opposite to the direct effect that we observed in the benchmark setting where more information leads to more production after this information becomes available. It turns out that, in our model, the strategic effect is always the (weakly) dominating one. This result is the next Lemma.

**Lemma 8** *In the no-commitment setting, more information increases production in period* 1, $\frac{\partial s}{\partial \lambda} \geq 0$.

**Proof.** See Appendix. ■

This implies that the set of symmetric equilibria expands to the right.

We summarize the main effects of more information in the next Proposition.

**Proposition 10** *More information (higher* $\lambda$*)*

*(i) decreases the cut-off type* $s$ *in case* $b$ *and shrinks the set of symmetric equilibria from the right in the benchmark setting;*

*(ii) expands the set of symmetric equilibria to the left in the full-commitment setting;*

*(iii) increases the cut-off type* $s$ *in case* $b$ *and expands the set of symmetric equilibria to the right in the no-commitment setting.*

The effects of higher $\lambda$ are opposite whether the agent is strategic or not. A higher $\lambda$ makes information obtained from the other firm more valuable. In the benchmark setting, this increases incentives of the principals to delay the production and to use information from the other firm. When the agent is strategic, this information can also be used for more efficient contracting in period 1, that is, even *before* it becomes available (strategic effect). In the no-commitment setting, this effects dominates. In the full-commitment setting, the take-it-or-leave-it offer made in period 1 does not depend on $\lambda$. However, whenever $s^A \leq s^B$, a higher $\lambda$ increases the chances that the principal $A$ will learn that the first–period offer was too low and will make a better offer in period 2. Then, while $s^A$ is not affected by $\lambda$ in both cases $a$ and $b$, profits in case $b$ increase with $\lambda$ and, therefore, the set of symmetric equilibria expands to the left ($\theta^{FC}$ falls).

## 5.2 Perfect learning

So far we have considered "contractual" learning, that is, the inference about the agents' costs made after in period 1 was based solely on the equilibrium contracts. Often, however, competitors may learn the production costs of a product doing reverse engineering. Then, each principal learns the exact cost of her agent if the other firm has produced in period 1. This is why we call it *perfect learning*. After no production in period 1, as before, the principal learns only that the agent's cost is above a certain cut-off. As in Section 2, consider the setup without competition.

Case $a$ does not change under perfect learning since, here, firm $A$ does not use the information revealed by firm $B$.

Consider case $b$. If agent $A$ does not accept producing at price $p_1$, principal $A$ will learn his type and, therefore, will offer $p_2^y = \theta$ in period 2. The agent will not get any rent and, thus, accepts any first-period price above his cost. That is, the cut-off type $s^A$ equals $p_1$ in all three settings. Even when strategic, the agent behaves "myopically" and the commitment ability of the principal no longer matters.

The problem of principal $A$ is now

$$\begin{cases} \max\limits_{s^A, p_2^n} \Pi_b = \left(v_1 - s^A\right) s^A + \delta \displaystyle\int_{s^A}^{s^B} \left(v_2 - \theta\right) d\theta + \delta \left(v_2 - p_2^n\right) \left(p_2^n - s^B\right) \\ \\ \text{s.t.} \quad s^A \leq s^B \leq p_2^n \leq 1. \end{cases}$$

Solving it, we obtain

$$s^A = \min\left\{\frac{v_1 - \delta v_2}{2-\delta}, s^B\right\}, \tag{14a}$$

$$p_2^n = \min\left\{\frac{v_2 + s^B}{2}, 1\right\}. \tag{14b}$$

Since in case $a$ perfect learning does not matter, our previous analysis applies for $s^B \leq \frac{v_1-\delta v_2}{2-\delta}$, in particular, the characterization of the cut-off type $s^A$ in Lemmas 1, 3 and 6. For $s^B > \frac{v_1-\delta v_2}{2-\delta}$, the three settings become identical and the same $s^A = p_1 = \frac{v_1-\delta v_2}{2-\delta}$ is optimal in case $b$. This implies that $\frac{v_1-\delta v_2}{2-\delta}$ becomes the upper end of the interval where $s^A = s^B$. Since $\frac{v_1-\delta v_2}{2-\delta} < \frac{v_1}{2}$, the sets of symmetric equilibria shrink in the full-commitment and no-commitment settings. In the benchmark setting, symmetric equilibria exist only if inter-period competition is present. For example, under inter-period competition from past sales, (14a) becomes

$$s^A = \min\left\{\frac{v_1 - \delta\gamma v_2}{2-\delta}, s^B\right\}$$

and $s^A = s^B$ if $s^B \in \left[\frac{v_1-\delta v_2+\delta}{2}, \frac{v_1-\delta\gamma v_2}{2-\delta}\right]$, which is the same interval as before (Lemma 4, case 0). Therefore, perfect learning does not affect the set of symmetric equilibria.[31,32]

The next Proposition summarizes the main effects of perfect learning.

**Proposition 11** *Introduction of perfect learning shrinks the sets of symmetric equilibria from the right in the full-commitment and no-commitment settings.*

Under perfect learning, the sets of symmetric equilibria still increase with the severity of the agency problem as before (see Proposition 7). However, this result is now driven by case $a$ where information is not used. In case $b$, the agency problem is the same across the three settings, and the sets of symmetric equilibria have the same upper endpoint.

[31] This is because at $s^B = \frac{v_1-\delta\gamma v_2}{2-\delta}$, when it becomes optimal to have $s^A < s^B$, the effect of perfect learning in terms of higher profits is still of the second order.

[32] In cases 1 and 2 of the benchmark setting, perfect learning decreases $\theta_1^{BM}$ and $\theta_2^{BM}$, respectively, since profits are higher under perfect learning in case $b$ while remaining unchanged in case $a$.

# 6 Conclusion

This paper investigated informational externalities across intra-firm bargaining games in a market setting. We found that the agency relationship creates a coordination motive in principals' actions making simultaneous production more likely. Each principal uses the information generated by the competitor to discipline her agent. This is done the best when the agents are put in the same conditions, that is, when they face the same incentives to produce. When the agency problem becomes more severe, there are more benefits in limiting the strategic behavior of the agents and the two firms tend to behave symmetrically even more. The effects of competition are very different depending whether it is intra- or inter-period. Inter-period competition penalizes firms when they produce in different periods and, therefore, it is conducive for the simultaneous production. Intra-period competition has an opposite effect since it pushes firms to differentiate and to produce in different periods.

This paper makes a step towards understanding the interactions between internal structure of firms and competition in an industry equilibrium. Competition plays a dual role since it affects both the costs and the benefits of solving the agency problem in any given way. The way the agency problem is solved then maps back into competition. We still have a very limited understanding of these interactions and general equilibrium effects, and more research is needed.

# Appendix

**Proof of Lemma 1.** In case $a$, transform (2) as follows. Assume that $p_2^n = 1$, solve for $p_1$ and find that $\frac{v_2+p_1}{2}$ is indeed larger than one. Then, assume that $p_2^n = \frac{v_2+p_1}{2}$ and find $p_1 = s^A$. This gives the following:

$$
\begin{array}{lll}
\textbf{Case 1.}\ \text{if } \frac{v_1+(2-\delta)v_2}{4-\delta} \geq 1: & & \\
\quad p_1 = \max\left\{\frac{v_1-\delta v_2+\delta}{2}, s^B\right\},\ p_2^n = 1, & & \\
\textbf{Case 2.}\ \text{if } \frac{v_1+(2-\delta)v_2}{4-\delta} < 1: & & \\
\quad p_1 = \frac{v_1-\frac{\delta}{2}v_2}{2-\frac{\delta}{2}},\ p_2^n = \frac{v_1+(2-\delta)v_2}{4-\delta}, & \text{if } s^B \leq \frac{v_1-\frac{\delta}{2}v_2}{2-\frac{\delta}{2}} & \\
\quad p_1 = s^B,\ p_2^n = \frac{v_2+s^B}{2}, & \text{if } s^B \in \left[\frac{v_1-\frac{\delta}{2}v_2}{2-\frac{\delta}{2}}, 2-v_2\right] & \\
\quad p_1 = s^B,\ p_2^n = 1, & \text{if } s^B \geq 2-v_2 &
\end{array}
\tag{15}
$$

In case $b$, transforming (4) in a similar way, we obtain that $p_2^n = \min\left\{\frac{v_2+s^B}{2}, 1\right\}$ and

$$\begin{array}{lll} p_1 = p_2^y = s^B, & \text{if } s^B \leq \frac{v_1-\delta v_2}{2-\delta} \\ p_1 = \frac{v_1-\delta v_2+\delta s^B}{2},\ p_2^y = s^B, & \text{if } s^B \in \left[\frac{v_1-\delta v_2}{2-\delta}, \frac{v_1+(2-\delta)v_2}{4-\delta}\right] \\ p_1 = \frac{v_1-\frac{\delta}{2}v_2}{2-\frac{\delta}{2}},\ p_2^y = \frac{v_1+(2-\delta)v_2}{4-\delta}, & \text{if } s^B \geq \frac{v_1+(2-\delta)v_2}{4-\delta} \end{array} \tag{16}$$

We are now left to determine where the change from case $a$ to case $b$ occurs. We will need the following Lemma.

**Lemma 9** *(i) If $p_1 = s^B$ in both cases, the profits are the same. (ii) If $p_1 = s^B$ in one of the two cases and $p_1 \neq s^B$ in the other, then profits are higher in the latter.*

***Proof.*** *(i) True by comparing (1) and (3) (for the latter case, $p_1 = p_2^y = s^B$). (ii) True since $p_1 = s^B$ is feasible in both cases.* ■

We will say that $p_1$ (and the corresponding case) is *constrained* when $p_1 = s^B$ in this case. If only one case is constrained, the other is optimal. The change from case $a$ to case $b$ occurs when either both are unconstrained, or both are constrained (so $p_1 = s^B$) and one case becomes unconstrained.

There are the following parameter ranges to consider.

**Case 1.** If $\frac{v_1+(2-\delta)v_2}{4-\delta} \geq 1$, up to $s^B = \frac{v_1-\delta v_2}{2-\delta}$ case $a$ is unconstrained while case $b$ is constrained. For $s^B \in \left(\frac{v_1-\delta v_2}{2-\delta}, \frac{v_1-\delta v_2+\delta}{2}\right)$ both cases are unconstrained and the cut-off $\theta_1^{BM}$ is found from the equality of $\Pi_a^{BM}$ and $\Pi_b^{BM}$, given by (1) and (3), respectively, with the corresponding prices. For $s^B > \theta_1^{BM}$, case $b$ is optimal.

**Case 2.** If $\frac{v_1+(2-\delta)v_2}{4-\delta} < 1$, then $\frac{v_1-\delta v_2}{2-\delta} < \frac{v_1-\frac{\delta}{2}v_2}{2-\frac{\delta}{2}}$, and therefore, up to $s^B = \frac{v_1-\delta v_2}{2-\delta}$ case $a$ is unconstrained while case $b$ is constrained; thus, case $a$ is optimal. For $s^B \in \left(\frac{v_1-\delta v_2}{2-\delta}, \frac{v_1-\frac{\delta}{2}v_2}{2-\frac{\delta}{2}}\right)$ both cases are unconstrained and $\theta_2^{BM}$ is found from the equality of (1) and (3) with the corresponding prices. For $s^B > \theta_2^{BM}$, case $b$ is optimal. ■

**Proposition 1** *In the benchmark setting, there are only two equilibria and they are asymmetric:*

1. *If $\frac{v_1+(2-\delta)v_2}{4-\delta} \geq 1$, in one firm the cut-off is $\frac{v_1-\delta v_2+\delta}{2}$ and in the other it is $\frac{(2+\delta)v_1-\delta v_2(2+\delta)+\delta^2}{4}$.*

2. *If $\frac{v_1+(2-\delta)v_2}{4-\delta} < 1$, in one firm the cut-off is $\frac{v_1-\frac{\delta}{2}v_2}{2-\frac{\delta}{2}}$ and in the other it is $\frac{\left(2+\frac{\delta}{2}\right)v_1-2\delta v_2}{4-\delta}$.*

**Proof.** Plug optimal second-period prices from (2) and (4) into (1) and (3), respectively. The profit function $\Pi^{BM}$

$$\Pi^{BM} = \left[ \begin{array}{l} \Pi_a^{BM}, \text{ if } s^A \leq s^B \\ \Pi_b^{BM}, \text{ if } s^A \geq s^B, \end{array} \right.$$

(where $\Pi_a^{BM}$ and $\Pi_b^{BM}$ are given by (1) and (3), respectively) is a function of $\left(s^A, s^B\right)$ only. Let us find the equilibria by construction.[33] Since symmetric pure-strategy equilibria do not exist (Corollary 1), we look for asymmetric ones. Then, one firm (say, $A$) will be in case $b$ while the other firm, firm $B$, will be in case $a$.

**Case 1:** $\frac{v_1+(2-\delta)v_2}{4-\delta} \geq 1$. If the firm $A$ is in case $b$, then its cut-off equals $s^{A*} = \frac{v_1-\delta v_2+\delta}{2}$ (see Lemma 1). Then, the reaction of firm $B$ is $s^{B*} = \frac{v_1-\delta v_2+\delta s^{A*}}{2}$ since $s^{A*} > \theta_1^{BM}$, that is,

$$s^{B*} = \frac{(2+\delta)\, v_1 - \delta v_2\, (2+\delta) + \delta^2}{4}.$$

For $\left(s^{A*}, s^{B*}\right)$ to be an equilibrium, firm $A$ must prefer to be indeed in case $b$, when firm $B$ has cut-off $s^{B*}$. This is equivalent to $s^{B*}$ being below $\theta_1^{BM}$. The necessary and sufficient condition for this is that, at $s^B = s^{B*}$, profits $\Pi_a^{BM}$ are higher than profits $\Pi_b^{BM}$.

Let us first find the optimal prices and then compare profits. Note that the following inequalities hold:

$$\frac{v_1-\delta v_2}{2-\delta} \leq s^{B*} \leq \frac{v_1-\delta v_2+\delta}{2} \leq \frac{v_1+(2-\delta)\, v_2}{4-\delta}.$$

Then, in the case where $p_1 \geq s^B$, $p_1 = \frac{v_1-\delta v_2+\delta}{2}$ and $p_2^n = 1$ (see (15)). If $p_1 \leq s^B$, then $p_1 = \frac{v_1-\delta v_2+\delta s^{B*}}{2}$, $p_2^y = s^{B*}$ and $p_2^n = \min\left\{\frac{v_2+s^{B*}}{2}, 1\right\}$ (see (16)). At $s^B = s^{B*}$, $\min\left\{\frac{v_2+s^{B*}}{2}, 1\right\} = 1$. Once these prices are plugged into $\Pi_a^{BM}$ and $\Pi_b^{BM}$, the difference in profits is positive:

$$\Pi_a^{BM}\mid_{s^B=s^{B*}} - \Pi_b^{BM}\mid_{s^B=s^{B*}} = \frac{1}{64}\delta^2(4-\delta^2)(v_1-\delta v_2+\delta-2)^2 > 0.$$

**Case 2:** $\frac{v_1+(2-\delta)v_2}{4-\delta} < 1$. If the firm $A$ is in case $b$, then its cut-off equals $s^{A*} = \frac{v_1-\frac{\delta}{2}v_2}{2-\frac{\delta}{2}}$ (see Lemma 1). Then, the reaction of firm $B$ is $s^{B*} = \frac{v_1-\delta v_2+\delta s^{A*}}{2}$ since $s^{A*} \in \left(\theta_2^{BM}, \frac{v_1+(2-\delta)v_2}{4-\delta}\right)$, that is,

$$s^{B*} = \frac{\left(2+\frac{\delta}{2}\right) v_1 - 2\delta v_2}{4-\delta}.$$

[33] Note that while $\Pi^{BM}$ is continuous in both arguments, it may not be quasiconcave in $s^A$(it may be M-shaped). Then, the standard theorem for the existence of a pure-strategy Nash equilibrium (Theorem 1.2 in Fudenberg and Tirole (1991) due to Debreu (1952)) cannot be applied.

For $\left(s^{A*}, s^{B*}\right)$ to be an equilibrium, firm $A$ must prefer to be indeed in case $b$, when firm $B$ has cut-off $s^{B*}$. This is equivalent to $s^{B*}$ being below $\theta_2^{BM}$. The necessary and sufficient condition for this is that, at $s^B = s^{B*}$, profits $\Pi_a^{BM}$ are higher than profits $\Pi_b^{BM}$.

Let us first find the optimal prices and then compare profits. Note that the following inequalities hold:

$$\frac{v_1 - \delta v_2}{2-\delta} \leq s^{B*} \leq \frac{v_1 - \frac{\delta}{2} v_2}{2 - \frac{\delta}{2}} \leq \frac{v_1 + (2-\delta)\, v_2}{4-\delta}.$$

Then, in the case where $s^A \geq s^B$, $p_1 = \frac{v_1 - \frac{\delta}{2}v_2}{2-\frac{\delta}{2}}$ and $p_2^n = \frac{v_1+(2-\delta)v_2}{4-\delta}$ (see (15)). If $s^A \leq s^B$, then $p_1 = \frac{v_1 - \delta v_2 + \delta s^{B*}}{2}$, $p_2^y = s^{B*}$ and $p_2^n = \min\left\{\frac{v_2+s^{B*}}{2}, 1\right\}$ (see (16)). At $s^B = s^{B*}$, $\min\left\{\frac{v_2+s^{B*}}{2}, 1\right\} = \frac{v_2+s^{B*}}{2}$. Once these prices are plugged into (1) and (3), the difference in profits is positive:

$$\Pi_a^{BM} \mid_{s^B = s^{B*}} - \Pi_b^{BM} \mid_{s^B = s^{B*}} = \frac{1}{16}\delta^2(4-\delta-\delta^2)(v_2-1)^2 \geq 0.$$

■

**Proof of Lemma 3.** Using Lemma 2, (6) becomes

$$\max_{p_1 \in [s^B, 1]} \Pi_a^{FC} = (v_1 - p_1)\, p_1$$

and the optimal price is $p_1 = \max\left\{\frac{v_1}{2}, s^B\right\}$. (8) becomes

$$\max_{p_1 \leq s^B \leq p_2^n \leq 1} \Pi_b^{FC} = (v_1 - p_1)\, p_1 + \delta\,(v_2 - p_2^n)\left(p_2^n - s^B\right)$$

and the optimal prices are $p_1 = s^A = \min\left\{\frac{v_1}{2}, s^B\right\}$ and $p_2^n = \min\left\{\frac{v_2+s^B}{2}, 1\right\}$.

Finally, $\Pi_a^{FC} < \Pi_b^{FC}$ for $s^B \geq \frac{v_1}{2}$, therefore, $\theta^{FC} < \frac{v_1}{2}$ and is found from the equality of profits in the two cases with the corresponding prices inserted. ■

**Lemma 10** *In the benchmark setting, under inter-period competition from past sales,*

*0. If* $\frac{v_1+(2-\delta)v_2}{2\gamma_p v_2 + 2 - \delta} \geq 1$, $s^A$ *is*

$$s^A = \left[\begin{array}{ll} \frac{v_1-\delta v_2+\delta}{2}, & \text{if } s^B \leq \frac{v_1-\delta v_2+\delta}{2} \\ s^B, & \text{if } s^B \in \left[\frac{v_1-\delta v_2+\delta}{2}, \frac{v_1-\delta\gamma_p v_2}{2-\delta}\right] \\ \frac{v_1-\delta\gamma_p v_2+\delta s^B}{2}, & \text{if } s^B \in \left[\frac{v_1-\delta\gamma_p v_2}{2-\delta}, \min\left\{\frac{v_1+(2-\delta)\gamma_p v_2}{4-\delta}, 1\right\}\right] \\ \frac{v_1-\frac{\delta}{2}\gamma_p v_2}{2-\frac{\delta}{2}}, & \text{if } s^B \geq \frac{v_1+(2-\delta)\gamma_p v_2}{4-\delta} \ \left(\text{if } \frac{v_1+(2-\delta)\gamma_p v_2}{4-\delta} < 1\right) \end{array}\right.$$

1. *If* $\frac{v_1+(2-\delta)v_2}{2\gamma_p v_2+2-\delta} < 1 \leq \frac{v_1+(2-\delta)v_2}{4-\delta}$*, there exists* $\theta_1^{BM} \in \left(\frac{v_1-\delta\gamma_p v_2}{2-\delta}, \frac{v_1-\delta v_2+\delta}{2}\right)$ *such that* $s^A$ *is*

$$s^A = \left[\begin{array}{ll} \frac{v_1-\delta v_2+\delta}{2}, & \text{if } s^B \leq \theta_1^{BM} \\ \frac{v_1-\delta\gamma_p v_2+\delta s^B}{2}, & \text{if } s^B \in \left[\theta_1^{BM}, \min\left\{\frac{v_1+(2-\delta)\gamma_p v_2}{4-\delta}, 1\right\}\right] \\ \frac{v_1-\frac{\delta}{2}\gamma_p v_2}{2-\frac{\delta}{2}}, & \text{if } s^B \geq \frac{v_1+(2-\delta)\gamma_p v_2}{4-\delta} \ (\text{if } \frac{v_1+(2-\delta)\gamma_p v_2}{4-\delta} < 1) \end{array}\right.$$

2. *If* $\frac{v_1+(2-\delta)v_2}{4-\delta} < 1$*, there exists* $\theta_2^{BM} \in \left(\frac{v_1-\delta\gamma_p v_2}{2-\delta}, \frac{v_1-\frac{\delta}{2}v_2}{2-\frac{\delta}{2}}\right)$ *such that* $s^A$ *is*

$$s^A = \left[\begin{array}{ll} \frac{v_1-\frac{\delta}{2}v_2}{2-\frac{\delta}{2}}, & \text{if } s^B \leq \theta_2^{BM} \\ \frac{v_1-\delta\gamma_p v_2+\delta s^B}{2}, & \text{if } s^B \in \left[\theta_2^{BM}, \frac{v_1+(2-\delta)\gamma_p v_2}{4-\delta}\right] \\ \frac{v_1-\frac{\delta}{2}\gamma_p v_2}{2-\frac{\delta}{2}}, & \text{if } s^B \geq \frac{v_1+(2-\delta)\gamma_p v_2}{4-\delta} \end{array}\right.$$

**Proof.** Proceeds in the same way as the proof of Lemma 1. ■

**Proof of Lemma 5.** The proof is based on the fact that $\Pi^{BM} \geq \Pi^{FC}$ since the full-commitment solution is feasible in the benchmark setting. The equality is reached if and only if all the prices and production are the same.

(i) In the benchmark setting, $p_1 = s^A$ and $p_2^y \in [s^A, s^B]$. When the two cut-offs are the same, then $p_1 = p_2^y = s^A = s^B$ (and $p_2^n$ is ex post optimal). In the full-commitment setting, $p_1 = p_2^y = s^A$ (Lemma 2). When the two cut-offs are the same, then $p_1 = p_2^y = s^A = s^B$ (and $p_2^n$ is ex post optimal). Thus, prices and production are the same in the two settings and profits are the same as well.

(ii) Suppose $s^A > s^B$ in the benchmark setting, then $p_2^n > s^A$. In the full-commitment setting, in order to have the same $s^A$, principal $A$ sets $p_1 = p_2^n = s^A$. The prices are different in the two settings and, therefore, the full-commitment profits are lower than the benchmark ones. If $s^A < s^B$, it is $p_2^y$ which is different in the two settings. If $s^A \neq s^B$ in the full-commitment setting, again, either $p_2^n$ or $p_2^y$ will be equal to $p_1$ while this is not the case in the benchmark setting. ■

**Proof of Lemma 6.** As before, there are two possible cases depending on whether $s^A$ is higher or lower than $s^B$. In each of these cases there are two subcases. We solve each subcase, write the combined solution for each case, and then characterize the (full) solution.

Start with case $a$, i.e., solving (10). There are two subcases. If $s^A \leq 2 - v_2$, then

$$\begin{aligned} p_1 &= \frac{(2-\delta)\, s^A + \delta v_2}{2} \\ p_2^n &= \frac{v_2 + s^A}{2}. \end{aligned}$$

We can reformulate the problem (10) as the one of choosing $s^A$

$$\max_{s^A \in [s^B, 2-v_2]} \frac{1}{2}\left[\left(2v_1 - (2-\delta)\, s^A - \delta v_2\right) s^A + \frac{1}{2}\delta \left(v_2 - s^A\right)^2\right].$$

The solution is

$$s^A = \min\left\{\max\left\{s^B, \frac{v_1 - \delta v_2}{2 - \frac{3}{2}\delta}\right\}, 2 - v_2\right\}.$$

If $s^A > 2 - v_2$,

$$\begin{aligned} p_1 &= (1-\delta)\, s^A + \delta \\ p_2^n &= 1. \end{aligned}$$

(10) becomes

$$\max_{s^A \in [\max\{2-v_2, s^B\}, 1]} \left(v_1 - (1-\delta)\, s^A - \delta\right) s^A + \delta\, (v_2 - 1)\left(1 - s^A\right)$$

and the solution is

$$s^A = \min\left\{\max\left\{s^B, \frac{v_1 - \delta v_2}{2\,(1-\delta)}, 2 - v_2\right\}, 1\right\}.$$

Combining the two subcases, the optimal $s^A$ is

$$s^A = \left[\begin{array}{ll} \min\left\{\max\left\{\frac{v_1-\delta v_2}{2(1-\delta)}, s^B\right\}, 1\right\}, & \text{if } 2 - v_2 \leq \frac{v_1-\delta v_2}{2-\frac{3}{2}\delta} \\ \min\left\{\max\left\{\frac{v_1-\delta v_2}{2(1-\delta)} \text{ or } \frac{v_1-\delta v_2}{2-\frac{3}{2}\delta}, s^B\right\}, 1\right\}, & \text{if } 2 - v_2 \in \left[\frac{v_1-\delta v_2}{2-\frac{3}{2}\delta}, \frac{v_1-\delta v_2}{2(1-\delta)}\right] \\ \min\left\{\max\left\{\frac{v_1-\delta v_2}{2-\frac{3}{2}\delta}, s^B\right\}, 1\right\}, & \text{if } 2 - v_2 \geq \frac{v_1-\delta v_2}{2(1-\delta)} \end{array}\right.$$

However, the intermediate case $2 - v_2 \in \left(\frac{v_1-\delta v_2}{2-\frac{3}{2}\delta}, \frac{v_1-\delta v_2}{2(1-\delta)}\right)$ is not a separate case. There is $\widetilde{v}_2 \in \left(\frac{v_1-\delta v_2}{2-\frac{3}{2}\delta}, \frac{v_1-\delta v_2}{2(1-\delta)}\right)$ such that

$$s^A = \left[\begin{array}{ll} \min\left\{\max\left\{\frac{v_1-\delta v_2}{2(1-\delta)}, s^B\right\}, 1\right\}, & \text{if } 2 - v_2 \leq \widetilde{v}_2 \\ \min\left\{\max\left\{\frac{v_1-\delta v_2}{2-\frac{3}{2}\delta}, s^B\right\}, 1\right\}, & \text{if } 2 - v_2 \geq \widetilde{v}_2 \end{array}\right. \tag{17}$$

Consider now case $b$, that is, solve (11). The second-period price when the other firm did not produce in period 1, $p_2^n$, is

$$p_2^n = \min\left\{\frac{v_2 + s^B}{2}, 1\right\}.$$

To find $s^A$, consider again two subcases. If $s^A \leq 2s^B - v_2$, the prices are

$$\begin{aligned} p_1 &= \frac{(2-\delta)\, s^A + \delta v_2}{2} \\ p_2^y &= \frac{v_2 + s^A}{2}. \end{aligned}$$

The maximization problem (11) becomes (omit the last part that depends only on $p_2^n$)

$$\max_{s^A \leq 2s^B - v_2} \frac{1}{2}\left[\left(2v_1 - (2-\delta)\, s^A - \delta v_2\right) s^A + \frac{1}{2}\delta \left(v_2 - s^A\right)^2\right] \tag{18}$$

and the solution is

$$s^A = \min\left\{\frac{v_1 - \delta v_2}{2 - \frac{3}{2}\delta}, 2s^B - v_2\right\}.$$

If $s^A \geq 2s^B - v_2$, the prices are

$$\begin{aligned} p_1 &= (1-\delta)\, s^A + \delta s^B \\ p_2^y &= s^B. \end{aligned}$$

The maximization problem (11) becomes (omit the last part that depends only on $p_2^n$)

$$\max_{s^A \in [2s^B - v_2, s^B]} \left(v_1 - (1-\delta)\, s^A - \delta s^B\right) s^A + \delta\left(v_2 - s^B\right)\left(s^B - s^A\right) \tag{19}$$

and the solution is

$$s^A = \max\left\{\min\left\{s^B, \frac{v_1 - \delta v_2}{2(1-\delta)}\right\}, 2s^B - v_2\right\}.$$

Combining the two subcases, there is $\theta_2^{NC}$ such that the optimal $s^A$ is

$$s^A = \left[\begin{array}{ll} s^B, & \text{if } s^B \leq \min\left\{\frac{v_1 - \delta v_2}{2(1-\delta)}, \theta_2^{NC}\right\} \\ \frac{v_1 - \delta v_2}{2(1-\delta)}, & \text{if } s^B \in \left[\frac{v_1 - \delta v_2}{2(1-\delta)}, \theta_2^{NC}\right] \text{ (when this region exists)} \\ \frac{v_1 - \delta v_2}{2 - \frac{3}{2}\delta}, & \text{if } s^B \geq \theta_2^{NC} \end{array}\right.$$

The cut-off $\theta_2^{NC}$ is found from the equality of (18) and (19) with the corresponding prices. If this equation does not have solutions, define $\theta_2^{NC}$ as being equal to one. Note that $s^A$ jumps down at $\theta_2^{NC}$.

Before we combine this case with the previous one, let us show that $\theta_2^{NC} > \frac{v_1}{2}$. To do this, we need that for $s^B \leq \frac{v_1}{2}$, profits (18) are smaller than profits (19). Consider (19). Since $\frac{v_1 - \delta v_2}{2(1-\delta)} \geq \frac{v_1}{2}$, $s^A = \max\left\{\min\left\{s^B, \frac{v_1 - \delta v_2}{2(1-\delta)}\right\}, 2s^B - v_2\right\} = \max\left\{s^B, 2s^B - v_2\right\} = s^B$. Therefore, $s^A = p_1 = p_2^y = s^B$. By Lemma 3 (and its proof), this is also the full-commitment solution for $p_1 \leq s^B$. Then, profits (19) cannot be lower than profits (18) and are actually strictly higher since, in the latter case, $s^A = \min\left\{\frac{v_1-\delta v_2}{2-\frac{3}{2}\delta}, 2s^B - v_2\right\} < s^B$.

Finally, we need to find out at which $s^B$ the change from case $a$ to case $b$ occurs. Denote it $\theta_1^{NC}$. For $s^B \leq \frac{v_1}{2}$, if $s^A \geq s^B$, the no-commitment solution (17) is different from the full-commitment solution (see Lemma 3 and its proof). Therefore, $\Pi_a^{NC} < \Pi_a^{FC}$. If $s^A \leq s^B$, then the full and no-commitment solutions coincide (see the paragraph above), and therefore, $\Pi_b^{NC} = \Pi_b^{FC}$. But then $\theta_1^{NC}$ has to be smaller than its full-commitment counterpart, $\theta^{FC}$. Note that

$$\theta_1^{NC} < \theta^{FC} < \frac{v_1}{2} < \theta_2^{NC}.$$

■

**Proof of Lemma 8.** The principal $A$'s problem is

$$\begin{cases} \max\limits_{p_1} \Pi_b^{NC} = (v_1 - p_1)\, s^A \\ +\delta \left[ \begin{array}{l} \lambda\left(\left(v_2 - p_2^{y,c}\right)\left(p_2^{y,c} - s^A\right) + \left(v_2 - p_2^{n,c}\right)\left(p_2^{n,c} - s^B\right)\right) \\ +\,(1-\lambda)\left(v_2 - p_2^i\right)\left(p_2^i - s^A\right) \end{array} \right] \\ \\ \text{s.t. } p_1 \leq s^B,\; p_2^{y,c} = \min\left\{\frac{v_2+s^A}{2}, s^B\right\},\; p_2^i = \min\left\{\frac{v_2+s^B}{2}, 1\right\}, \\ p_2^i = \min\left\{\frac{v_2+s^A}{2}, 1\right\},\; (13) \end{cases}$$

Replace $p_1 = s^A(1-\delta) + \delta\left[\lambda p_2^{y,c} + (1-\lambda)\right]$ so that the problem amounts to the choice of $s^A$. We need to show that $\frac{\partial^2 \Pi_b^{NC}}{\partial\lambda\partial s^A} \geq 0$. Differentiate $\Pi_b^{NC}$ with respect to $\lambda$

$$\frac{\partial \Pi_b^{NC}}{\partial \lambda}\frac{1}{\delta} = \left(v_2 - p_2^{n,c}\right)\left(p_2^{n,c} - s^B\right) + \left(p_2^i - p_2^{y,c}\right)\left(p_2^i + p_2^{y,c} - v_2\right).$$

Then,

$$\frac{\partial^2 \Pi_b^{NC}}{\partial\lambda\partial s^A}\frac{1}{\delta} = \left[\begin{array}{ll} 0, & \text{if } p_2^{y,c} = p_2^i = \frac{v_2+s^A}{2} \\ \frac{s^A}{2} & \text{if } p_2^{y,c} = s^B,\; p_2^i = \frac{v_2+s^A}{2} \\ 0, & \text{if } p_2^{y,c} = s^B,\; p_2^i = 1 \end{array}\right.$$

■

# References


Boyer, M., and J.-J. Laffont (2003): "Competition and the reform of incentive schemes in the regulated sector," *Journal of Public Economics*, 87(9-10), 2369–2396.

Crémer, J., and R. P. McLean (1985): "Optimal Selling Strategies under Uncertainty for a Discriminating Monopolist when Demands are Interdependent," *Econometrica*, 53(2), 345–361.

De Bondt, R. (1997): "Spillovers and innovative activities," *International Journal of Industrial Organization*, 15(1), 1–28.

Décamps, J.-P., and T. Mariotti (2004): "Investment timing and learning externalities," *Journal of Economic Theory*, 118(1), 80–102.

Demougin, D. M., and D. A. Garvie (1991): "Contractual Design with Correlated Information under Limited Liability," *RAND Journal of Economics*, 22(4), 477–489.

Dixit, A., and R. Pindyck (1994): *Investment under Uncertainty*. Princeton University Press.

Drugov, M. (2007): "Contracts with Informational Externalities," University of Oxford, Department of Economics, mimeo.

——— (2010): "Information and Delay in an Agency Model," *RAND Journal of Economics*, 41(3), 598–615.

Fuchs, W., and A. Skrzypacz (2010): "Bargaining with Arrival of New Traders," *American Economic Review*, 100(3), 802–36.

Fudenberg, D., and J. Tirole (1991): *Game Theory*. MIT Press.

Gary-Bobo, R., and Y. Speigel (2006): "Optimal State-Contingent Regulation Under Limited Liability," *The RAND Journal of Economics*, 37(2), 430–447.

Golan, L., C. A. Parlour, and U. Rajan (2007): "Racing to the Bottom: Competition and Quality," mimeo.

Gu, W., and P. Kuhn (1998): "A Theory of Holdouts in Wage Bargaining," *The American Economic Review*, 88(3), 428–449.

Hart, O., and J. Tirole (1988): "Contract Renegotiation and Coasian Dynamics," *The Review of Economic Studies*, 55, 509–540.

Holmström, B. (1982): "Moral Hazard in Teams," *The Bell Journal of Economics*, 13(2), 324–340.

Kuhn, P., and W. Gu (1999): "Learning in Sequential Wage Negotiations: Theory and Evidence," *Journal of Labor Economics*, 17(1), 109–140.

Lazear, E. P., and S. Rosen (1981): "Rank-Order Tournaments as Optimum Labor Contracts," *The Journal of Political Economy*, 89(5), 841–864.

Marin, D., and T. A. Verdier (2008): "Power Inside The Firm and The Market: A General Equilibrium Approach," *Journal of the European Economic Association*, 6(4), 752–788.

Meyer, M. A., and J. Vickers (1997): "Performance Comparisons and Dynamic Incentives," *The Journal of Political Economy*, 105(3), 547–581.

Nalebuff, B. J., and J. E. Stiglitz (1983): "Information, Competition, and Markets," *The American Economic Review*, 73(2), 278–283.

Raith, M. (2003): "Competition, Risk, and Managerial Incentives," *The American Economic Review*, 93(4), 1425–1436.

Reinganum, J. F. (1989): "Chapter 14 The timing of innovation: Research, development, and diffusion," in *Handbook of Industrial Organization*, ed. by R. Schmalensee, and R. Willig, vol. 1, pp. 849–908. Elsevier.

Riordan, M. H., and D. E. M. Sappington (1988): "Optimal contracts with public ex post information," *Journal of Economic Theory*, 45(1), 189–199.

Rob, R. (1991): "Learning and Capacity Expansion under Demand Uncertainty," *The Review of Economic Studies*, 58(4), 655–675.

Shleifer, A. (1985): "A Theory of Yardstick Competition," *The RAND Journal of Economics*, 16(3), 319–327.

Sobel, J., and I. Takahashi (1983): "A Multistage Model of Bargaining," *The Review of Economic Studies*, 50(3), 411–426.

Strausz, R. (2006): "Interim Information in Long-Term Contracts," *Journal of Economics and Management Strategy*, 15(4), 1041–1067.